GRADUATE SCHOOL OF MANAGEMENT
FACULTY OF APPLIED MATHEMATICS & CONTROL PROCESSES
St. Petersburg University
THE INTERNATIONAL SOCIETY OF DYNAMIC GAMES
(Russian Chapter)

GAME THEORY AND MANAGEMENT

The Fourth International Conference
Game Theory and Management

GTM2010

June 28-30, 2010, St. Petersburg, Russia

ABSTRACTS

Edited by Leon A. Petrosyan and Nikolay A. Zenkevich

Graduate School of Management
St. Petersburg University
St. Petersburg
2010

УДК 518.9, 517.9, 681.3.07

GAME THEORY AND MANAGEMENT. Collected abstracts of papers presented on the Fourth International Conference Game Theory and Management / Editors Leon A. Petrosyan, Nikolay A. Zenkevich. – SPb.: Graduate School of Management SPbU, 2010. – 264 p.

The collection contains abstracts of papers accepted for the International Conference Game Theory and Management (June 28–30, 2010, St. Petersburg University, Russia). The presented abstracts belong to the field of game theory and its applications to management.

The abstract volume may be recommended for researches and post-graduate students of management, economic and applied mathematics departments.

Computer design: Anna V. Iljina, Andrew V. Zyatchin

ТЕОРИЯ ИГР И МЕНЕДЖМЕНТ. Сб. тезисов 4-й международной конференции по теории игр и менеджменту / Под ред. Л. А Петросяна, Н. А. Зенкевича. – СПб.: Высшая школа менеджмента СПбГУ, 2010. – 264 с.

Сборник содержит тезисы докладов участников 4-й международной конференции по теории игр и менеджменту (28–30 июня 2010 года, Высшая школа менеджмента, Санкт-Петербургский государственный университет, Россия). Представленные тезисы относятся к теории игр и её приложениям в менеджменте.

Тезисы представляют интерес для научных работников, аспирантов и студентов старших курсов университетов, специализирующихся по менеджменту, экономике и прикладной математике.

Компьютерная верстка: А.В. Ильина, А.В. Зятчин

CONTENTS

Plenary Speakers

Alain Haurie	University of Geneva (Switzerland)
Arkady Kryazhimskiy	International Institute for Applied Systems Analysis (Austria) and Steklov Mathematical Institute RAS (Russia)
Hervé Moulin	Rice University (USA)
Ralph Tyrrell Rockafellar	University of Washington (USA)

Organizing Committee

Valery Katkalo – co-chair	GSOM, St. Petersburg University, Russia
Leon Petrosyan – co-chair	FAM, CGT, St. Petersburg University, Russia
Nikolay Zenkevich - deputy	GSOM, CGT, St. Petersburg University, Russia
Maria Dorokhina	GSOM, St. Petersburg University, Russia
Margarita Gladkova	GSOM, St. Petersburg University, Russia
Varvara Gorbunova	GSOM, St. Petersburg University, Russia
Maria Yurova	GSOM, St. Petersburg University, Russia
Andrew Zyatchin	GSOM, St. Petersburg University, Russia

International Program Committee

Eitan Altman	INRIA, France
Jesus Mario Bilbao	University of Seville, Spain
Andrea Di Liddo	University of Foggia, Italy
Irinel Dragan	University of Texas, USA
Hongway Gao	Qingdao University, China
Andrey Garnaev	St. Petersburg University, Russia
Sergiu Hart	Hebrew University, Israel
Steffen Jørgensen	University of Southern Denmark, Denmark
Ehud Kalai	Northwestern University, USA
Valery Katkalo – co-chair	St. Petersburg University, Russia
Vladimir Mazalov	Institute of Applied Mathematical Research, Russia
Shigeo Muto	Tokyo Institute of Technology, Japan
Leon Petrosyan – co-chair	St. Petersburg University, Russia
Richard Staelin	Duke University, USA
Krzysztof J. Szajowski	Institute of Mathematics and Computer Science, Poland
Myrna Wooders	Vanderbilt University, USA
David W.K.Yeung	Hong Kong Baptist University, Hong-Kong
Georges Zaccour	HEC, Canada
Nikolay Zenkevich	Saint-Petersburg University, Russia
Paul Zipkin	Duke University, USA

WELCOME ADDRESS

We are pleased to welcome you at the Fourth International Conference on Game Theory and Management (GTM2010) which has been organized by the Graduate School of Management (GSOM), St. Petersburg University in collaboration with the Faculty of Applied Mathematics - Control Processes and with the International Society of Dynamic Games (Russian Chapter).

The Conference is designed to support further development of dialogue between fundamental game theory research and advanced studies in management. Such collaboration had already proved to be very fruitful, and has been manifested in the last two decades by Nobel Prizes in Economics awarded to John Nash, John Harsanyi, Reinhard Selten, Robert Aumann, Eric Maskin, Roger Mayerson and few other leading scholars in game theory. In its applications to management topics game theory contributed in very significant ways to enhancement of our understanding of the most complex issues in competitive strategy, industrial organization and operations management, to name a few areas.

Needless to say that Game Theory and Management is very natural area to be developed in the multidisciplinary environment of St. Petersburg University which is the oldest (est. 1724) Russian classical research University. This Conference was initiated in 2006 at SPbU as part of the strategic partnership of its GSOM and the Faculty of Applied Mathematics - Control Processes, both in internationally recognized centers of research and teaching.

We would like to express our gratitude to the Conference's key speakers – distinguished scholars with path-breaking contributions to economic theory, game theory and management – for accepting our invitations. We would also like to thank all the participants who have generously provided their research papers for this important event. We are pleased that this Conference has already become a tradition and wish all the success and solid worldwide recognition.

Co-chairs GTM2010

Professor Valery S. Katkalo,
Dean, Graduate School of Management

Professor Leon A. Petrosyan
Dean, Faculty of Applied
Mathematics & Control Processes

St. Petersburg University

WELCOME

On behalf of the Organizing and Program Committees of GTM2010, it gives us much pleasure to welcome you to the International Conference on Game Theory and Management in the Graduate School of Management and Faculty of Applied Mathematics - Control Processes of St. Petersburg University. This conference is the third of the St. Petersburg master-plan conferences on Game Theory and Management, the first one of which took place also in this city three years before. It is an innovated edition as to investigate the trend and provide a unique platform for synergy among business and financial systems, on one hand and industrial systems, on the other, in game-theoretic support of national economies in the recent process of globalization. Mathematical and especially game-theoretic modeling the globalized systemic structure of the world of the future, and managing its conduct towards common benefits is becoming a primary goal today.

This conference held in new millennium is not unique as the fourth international conference on Game Theory and Management since after the conferences GTM2007, GTM2008, GTM2009 other international workshops on Dynamic Games and Management were held in Montreal Canada. Because of the importance of the topic we hope that other international and national events dedicated to it will follow. Starting our activity in this direction three years before we had in mind that St. Petersburg University was the first university in the former Soviet Union where game theory was included in the program as obligatory course and the first place in Russia where Graduate School of Management and Faculty of Applied Mathematics were established.

The present volume contains abstracts accepted for the Fourth International Conference on Game Theory and Management, held in St. Petersburg, June 28-30, 2010. As editors of the Volume IV of Contributions to Game Theory and Management we invite the participants to present their full papers for the publication in this Volume. By arrangements with the editors of the international periodical Game Theory and Applications the conference may recommend the most interesting papers for publication in this journal.

St. Petersburg is especially appropriate as a venue for this meeting, being “window to Europe” and thus bridging the cultures of East and West, North and South.

Acknowledgements. The Program and Organizing Committees thanks all people without whose help this conference would not have been possible: the invited

speakers, the authors of papers all of the members of Program Committee for referring papers, the staffs of Graduate School of Management and Faculty of Applied Mathematics - Control Processes.

We would like to thank Maria Dorokhina, Margarita Gladkova, Anna Iljina, Andrey Zyatchin and Maria Yurova for their effective efforts in preparing the conference.

We thank them all.

Leon A. Petrosyan, GTM2010 Program Committee
Nikolay A. Zenkevich, GTM2010 Organizing Committee

Periodicals in Game Theory

GAME THEORY AND APPLICATIONS

Volumes 1–14

Edited by
Leon A. Petrosyan & Vladimir V. Mazalov

NOVA SCIENCE

Graph Searching Games with a Radius of Capture

Tatiana Abramovskaya[1] and **Nikolai Petrov**[2]

[1,2]*St. Petersburg University*
Russia
[1]*tanya.abramovskaya@gmail.com*
[2]*nikolai.petrov@pobox.spbu.ru*

Keywords: *Guaranteed search, Search numbers, Radius of capture, Golovach function, Unit jumps*

The problem under investigation belongs to pursuit-evasion differential games theory on graphs with no information on the evader. A team of pursuers tries to catch an evader who is "invisible" for those. The search problem is to determine for each graph the minimum number of pursuers needed to capture an evader. Such problems attach the attention of many researchers because of their connections with different problems of Graph theory. Detailed annotated bibliography on guaranteed graph searching is given in [1].

Golovach stated a search problem called ε-search problem (see [2]). The goal of the team of pursuers is construction of the program which ensures the approach of the evader and one of the pursuers within distance (in the inner metric of graph) less than or equal to the given nonnegative number ε called the radius of capture. The search problem is to determine for each topological graph ε-search number, that is, the minimum number of pursuers needed to catch the evader.

We name the function which associates each nonnegative number ε with ε-search number as Golovach function. It is known that Golovach function is piecewise constant, non-increasing and right-continuous. The next property of Golovach function regards to its jumps.

Let G be a graph and k be the ε-search number of G. We say that there is a jump of height h of Golovach function at ε if there exists $\varepsilon' < \varepsilon$ such that for each δ, $\varepsilon' < \delta < \varepsilon$, a team of $k+h$ pursuers succeed in ε-search whereas team of $k+h-1$ pursuers can't do it.

As it was proved by Golovach and Petrov in [3] Golovach function for a complete graph with more than 5 nodes may have non-unit jumps. The authors are also aware of certain examples of trees that disprove the conjecture that Golovach function for any planar graph has only unit jumps. Nevertheless, a subclass of trees for which the Golovach function has only unit jumps is specified.

The given counter-examples have the minimum number of edges. The first counter-example consists of 28 edges (it has a vertex with degree 4). The assertion that Golovach function for trees consisting of no more than 27 edges has only unit jumps is true. The second counter-example consists of 29 edges and degrees of its vertices are no more than 3. The assertion that Golovach function for trees containing no more than 28 edges and vertices with degrees ≤ 3 has only unit jumps is true.

The lengths of edges play a significant role in characterization of subclasses of trees with non-degenerate Golovach function. We can ask about a graph with degenerate Golovach function: do exist arbitrarily small variations of lengths of its edges such that Golovach function for "deformed" graphs has no non-unit jumps. The authors constructed appropriate variations for counter-examples mentioned above.

References

[1] F. V. Fomin, D. M. Thilikos, An annotated bibliography on guaranteed graph searching, Theoretical Computer Science, 399 (2008), pp. 236 –245.

[2] P. A. Golovach, An extremal search problem on graphs (Russian), Vestnik Leningrad. Univ. Mat. Mekh. Astronom., (1990), pp. 16 –21. translation in Vestnik Leningrad. Univ. Math. 23 (1990), no. 3, 19 –25.

[3] P. A. Golovach, N. N. Petrov, The search number of a complete graph, Vestnik Leningrad. Univ. Mat. Mekh. Astronom., (1986), pp. 57 –60.

Dynamics and Stability of Constitutions, Coalitions, and Clubs

Daron Acemoglu[1], **Georgy Egorov**[2] **and Konstantin Sonin**[3]

[1]*MIT,*
United States

[2]*Northwestern University,*
United States

[3]*New Economic School,,*
Russia

Keywords*: Commitment, Dynamic coalition formation, Political economy, Voting*

A central feature of dynamic collective decision-making is that the rules that govern the procedures for future decision-making and the distribution of political power across players are determined by current decisions. For example, current constitutional change must take into account how the new constitution may pave the way for further changes in laws and regulations. We develop a general framework for the analysis of this class of dynamic problems. Under relatively natural acyclicity assumptions, we provide a complete characterization of dynamically stable states as functions of the initial state and determine conditions for their uniqueness. We show how this framework can be applied in political economy, coalition formation, and the analysis of the dynamics of clubs. The explicit characterization we provide highlights two intuitive features of dynamic collective decision-making: (1) a social arrangement is made stable by the instability of alternative arrangements that are preferred by sufficiently many members of the society; (2) efficiency-enhancing changes are often resisted because of further social changes that they will engender.

The Incompatibility Banzhaf Value

José Alonso-Meijide[1], **Mikel Álvarez-Mozos**[2] and
Gloria Fiestras-Janeiro[3]

Department of Statistics and Operations Research,
University of Santiago de Compostela,
Spain
[1]*Faculty of Sciences of Lugo*
[2]*Faculty Mathematics*
mikel.alvarez@usc.es
[3]*Faculty of Economics*

Keywords: *Cooperative games, Banzhaf value, Incompatibilities, Axiomatic characterization*

The classical Transferable Utility cooperative game (TU Game from now on) is a model in which agents bargain and are able to make binding agreements, and hence, players may enforce cooperative behaviour and form coalitions. The structure of the game is described by means of a characteristic function which is an assessment of the profit that each coalition can obtain by its own. It is also assumed that the joint profits can be divided and transferred among the members of the coalition. However, there are many real-life situations in which not all coalitions are possible.

Therefore, there have been several proposals to extend the original model to a more general one in which not all coalitions are possible. We will call such models TU games with restricted cooperation. The first such attempt was the model of games with coalition structure proposed in Aumann and Drèze (1974) where a partition of the set of players is assumed to be given describing blocks of players that will form and bargain for the division of the worth of the grand coalition. The second and one of the most studied proposals so far is the model of TU games with graph restricted communication proposed in Myerson (1977). Here, it is assumed that players can only communicate, and hence, cooperate using a given communication graph. In Bergantiños et al. (1993) TU games with incompatibilities are introduced. In this model it is assumed that an incompatibility graph is given which describes the pairs of players which are incompatible, and hence, they can not cooperate at all. Another way to define a TU game with restricted cooperation was proposed in Gilles et al. (1992). The so called games with permission structure assume the existence of an ordering of the set of players which describes when is each player able to join a coalition. The model of games on augmenting systems proposed in Bilbao (2003) is an interesting way to extend some models of games with restricted cooperation described above.

One of the most interesting questions in these contexts is how to allocate the joint profit generated from the cooperation. One of the most important answers to it was given in Shapley (1953) and it is known as the Shapley value. Another interesting value was introduced by Banzhaf (1965), initially proposed in the context of voting games and later on extended to general TU games by Owen (1975). A player's assessment by means of these values is the average of his marginal contribution to any coalition to which the player does not belong. But they differ in the weights associated to each marginal contribution. The main difference between these two values is that the Shapley value is efficient, while the Banzhaf value satisfies the total power property. Both the Shapley and the Banzhaf value have been extended to different models of games with restricted cooperation. In Owen (1977) the Shapley value is extended to games with coalition structure and in Owen (1982) and Alonso-Meijide and Fiestras-Janeiro (2002) the Banzhaf and Shapley values are extended to games with coalition structure. For games with graph restricted cooperation Myerson (1977) proposed an extension of the Shapley value and Alonso-Meijide and Fiestras-Janeiro (2006) proposed an extension of the Banzhaf value.

Bergantiños et al. (1993) introduced the games with incompatibilities and proposed and characterized a modification of the Shapley value for such situations. In Alon so-Meijide and Casas-Méndez (2007) a modification of the Public Good Index for simple games with incompatibilities was proposed. In this paper, we define a modification of the Banzhaf value for this kind of situations and provide two characterizations of it. Comparing the characterizations of the Incompatibility Shapley value and the Incompatibility Banzhaf value we see that the differences between the original Shapley and Banzhaf values are transferred to this setting.

Finally, we illustrate the proposed solution concept with a real world example. Since the Banzhaf value is of particular interest when we come to study the decisiveness of agents in a voting body, the proposed example is the Basque Country Parliament in its III term of office.

Referencs

[1]. Aumann, R.J. and Drèze, J. (1974). Cooperative games with coalition structures. International Journal of Game Theory. 3. 217-237.

[2]. Alonso-Meijide, J. M. and Casas-Méndez, B. (2007). The Public Good Index when some voters are incompatible, Homo Oeconomicus. 24. 449-468.

[3]. Alonso-Meijide, J.M. and Fiestras-Janeiro, M.G. (2002). Modification of the Banzhaf value for games with coalition structure. Annals of Operations Research. 109. 213-227.

[4]. Alonso-Meijide, J.M. and Fiestras-Janeiro, M.G. (2006). The Banzhaf value and communication situations. Naval Research Logistics. 53. 198-203.

[5]. Banzhaf, J.F. (1965). Weighted voting does not work: A mathematical analysis. Rutgers Law Review. 19. 317-343.

[6]. Bergantiños, G., Carreras, F., and García-Jurado, I. (1993). Cooperation when some players are incompatible. Zeitschrift für Operations Research. 38. 187-201.

[7]. Bilbao, J.M. (2003). Cooperative games under augmenting systems. SIAM Journal of Discrete Mathematics. 5. 305-320.

[8]. Gillies, R.P., Owen, G. y Brink, R. (1992). Games with permission structures: The conjunctive approach. International Journal of Game Theory. 20. 277-293.

[9]. Myerson, R.B. (1977). Graphs and cooperation in games. Mathematics of Operations Research. 2. 225-229.

[10]. Owen, G. (1975). Multilinear Extensions and the Banzhaf value. Naval research logistics quarterly. 22. 741-750.

[12]. Owen, G. (1977). Values of games with a priori unions, in R. Henn and O. Moeschlin (eds). 'Mathematical Economics and Game Theory'. Springer. 76-88.

[13]. Owen, G. (1986). Values of graph restricted games. SIAM Journal on Algebraic and Discrete Methods. 7. 210-220.

Finitely Repeated Bilateral Trade with Renegotiation

Georgy Artemov[1], **Sergei Guriev**[2] and **Dmitriy Kvasov**[3]

[1]*The University of Melbourne,*
Australia
gartemov@unimelb.edu.au

[2]*New School of Economics,*
Russia

[3]*The University of Adelaide,*
Australia

Keywords: *Dynamic mechanisms, Renegotiatioin, Bilateral trade*

We study a bilateral trade problem that is repeated finitely many times. In each period seller may sell to a buyer a unit of indivisible good; the valuations for the goods are independent both across agents and across periods. We assume that the budget needs to be balanced in every period. After each period, either player can refuse the exchange; thus, we impose *per-period ex-post* IR constraints. In the last period of relationship, imposing We study a bilateral trade problem that is repeated nitely many times.In each period seller may sell to a buyer a unit of indivisible good; the valuations for the goods are independent both across agents and across periods.We assume that the budget needs to be balanced in every period. We assume that the agents' announcements can be made simultaneous and the terms of trade are non-negotiable once the announcement is made, hence we impose interim IC constraints. After each period, either player can refuse the exchange; thus, we impose per-period ex-post IR constraints. The relationship breaks down exogenously after a (commonly known) number of repetitions. In the last period of relationship, imposing budget balance, IC and IR leads to inecient trade, as described by Myerson and Satterthwaite (1983) and Gresik (1991). However, in any period but last, the agents value future relationship. The ex-ante surplus that this relationship generates enters per-period ex-post IR constraint, thus relaxing it and allowing for more trade in each round but the last, as compared to static bound found by Myerson and Satterthwaite. We show that if the relationship lasts long enough, trade in the rst periods is fully ecient. We thus can conclude that, as the number of periods gets large, trade

approaches the fully ecient level and the speed of convergence is exponential. Our result does not rely on the assumption that, if an agent deviates and does not follow the prescription of the mechanism, her future surplus is set to zero. In fact, we assume that agents are able to return to the equilibrium path in case of the deviation by the start of the next period. However, the mechanism sets a disagreement point for the deviation: it prescribes that the continuation play after the deviation is take-it-or-leave-it offers, made by the party who has not deviated, until the end of the relationship. As exante surplus of take-it-or-leave-it offers is higher than ex-ante surplus of anecient mechanism for the party making offers (Williams 1987), that party needs to be compensated to return to the equilibrium path (We assume a worst-case scenario that the deviator holds all the bargaining power as the parties divide an extra total surplus generated by moving from take-it-or-leave-it offers to an ecient mechanism, yet the party that is to make take-it-or-leave-it offers would need to be compensated for the lost surplus. Note that they are to agree to switch back to the equilibrium path before the beginning of the next period. So, there is no incomplete information at this stage.). This compensation gives us the ex-ante surplus that would enter into per-period ex-post IR constraint and allow to generate higher levels of trade. The relaxation of IR constraint is akin a subsidy discussed by Myerson and Satterthwaite; thus, it is a straightforward extension of their results to show that relaxed IR leads to higher levels of trade. We show how the relaxation comes naturally when relationship is repeated.

References

[1] Gresik, Thomas A., (1991) "Ex ante ecient, ex post individually rational trade," Journal of Economic Theory, Elsevier, vol. 53(1), pages 131-145, February,

[2] Myerson, Roger B. and Satterthwaite, Mark A., (1983) "Ecient mechanisms for bilateral trading," Journal of Economic Theory, Elsevier, vol. 29(2),pages 265-281, April,

[3] Williams, Steven R., (1987) "Ecient performance in two agent bargaining," Journal of Economic Theory, Elsevier, vol. 41(1), pages 154-172,February.

Bidder-Optimal Signal Structure in a First-Price Auction

Helmuts Azacis[1] and **Peter Vida**[2]

[1]*Cardiff University,*
United Kingdom
azacish@cf.ac.uk

[2]*University of Vienna,*
Austria
peter.vida@univie.ac.at

Keywords: *First price auction, Revenue equivalence, Multidimensional auctions, Bidder-optimal, Signal structure, Collusion*

We study optimal bidder collusion at first-price auctions when the collusive mechanism only relies on signals about bidders' valuations. We build on Fang and Morris [Fang, Hanming and Morris, Stephen, 2006. Multidimensional private value auctions. Journal of Economic Theory 126, 1-30] when two bidders have low or high private valuation of a single object and additionally observe a noisy signal about their opponents' valuations. We investigate the general case when the signals are chosen independently and identically out of $n \geq 2$ possible values. We demonstrate the unique symmetric equilibrium of the first price auction. We characterize the signal structure which provides the least revenue for the seller for arbitrary n. As a corollary, we show that bidders are *strictly* better off as signals can take on more and more possible values. Nevertheless, we provide an example which shows that the seller's revenue drops below the above optima even with only 2, but correlated signals.

JEL Classification Numbers: D44:D82

Purified Folk Theorem with Heterogeneous Time Preferences

Romeo Balanquit

Jawaharlal Nehru University,
India
rtbalanquit@yahoo.com

Keywords: *Repeated games, Folk Theorem, Pure strategies*

Abstract: *The result that the Folk Theorem can also be achieved by only employing pure strategies (Fudenberg and Maskin, 1991) is extended in this study where players have heterogeneous discount factors. It is shown as a consequence that this diversity in time preferences among players provides a larger feasible set of perfect equilibrium payoffs. Lehrer and Pauzner (1999) showed, under integrable continuous-time payoff stream for two players, that this set is indeed bigger than the perfect equilibrium set generated within the typical convex hull of a single-stage game. While this paper, on the other hand, maintains the use of the normal discrete-time framework, it generalizes Lehrer and Pauzner's result to n players and so as the Folk Theorem to a repeated game with heterogeneous time preferences.*

Fudenberg and Maskin (1991, henceforth F&M) showed that it is possible for the Folk Theorem to be sustained even without public randomization. They proved this by showing that in a game with very patient players, any correlated mixed action can be represented by a cyclical combination of pure strategy actions with appropriate frequency. In other words, whatever feasible payoffs players could obtain in the long-run by throwing a die could also be generated when they simply program their actions over time in a recurring fashion. A very important assumption in their framework is that players have common discount factor and that this is very close to one.

Now suppose we allow the discount factor to vary across players while maintaining them to be sufficiently high. The normal intuition is that it will fail to achieve the result of F&M since some players, particularly the less patient ones, may not have the incentive to remain in an infinitely-repeated game. However, Lehrer and Pauzner (L&P) argued that when there is difference in the discount factors between two players, a strategy can be sustained in equilibrium in an infinitely repeated game by trading player's payoffs overtime. This means that the more patient fellow could allow the less patient one to earn higher payoff during the beginning stages provided that she (the more patient one) be reciprocated to earn higher payoff towards the latter part of the game. While this set-up is shown to be subgame-ε-perfect and beneficial to both players, it is also shown that this intertemporal trading of payoffs could even broaden the possibilities for stable cooperation.

To ensure that this intertemporal trading can target any payoff (i.e. average discounted income over infinite time) within the convex hull of the set of pure-strategy actions, L&P makes use of a continuous-time payoff stream for two players. We show in this paper that this is not necessary and the typical use of discrete stages is sufficient to achieve this result and which can even be generalized to n players. Ultimately, this objective shall lead us to a generalization of pure-strategy Folk theorem with differentiated discount factors.

The main challenge in pursuing this goal is to convexify or "fill-up the inside" of the feasible space generated by the pure-strategy actions of n players. Our main ingredient for achieving this is a generalization of an F&M result which can be restated that for all discount factors sufficiently high, every payoff within the feasible space can be represented by the discounted average of a deterministic sequence of pure-strategy payoffs. Other subsequent step to our proof, for example, is to select only those payoffs that are individually rational.

References

[1] Abreu, D. (1988), "On the Theory of Infinitely Repeated Games with Discounting", Econometrica, vol.56, no. 2, pp.383-396.

[2] Fudenberg, D. and E. Maskin (1986), "The Folk Theorem in Repeated Games with Discounting or with Complete Information", Econometrica, vol. 54, no.3, pp.533-554.

[3] Fudenberg, Drew & Maskin, Eric (1991), "On the Dispensability of Public Randomization in Discounted Repeated Games". Journal of Economic Theory, 53(2):428-438.

[4] Lehrer, E. and A. Pauzner (1999), "Repeated Games with Differential Time Preferences", Econometrica, vol. 67, no.2, pp.393-412.

On Existence and Uniqueness of Perfect Equilibria in Stochastic Economies of Partial Commitment with Capital and Labor

Lukasz Balbus[1], **Kevin Reffet**[2] and **Łukasz Woźny**[3]

[1]*Wroclaw University of Technology*,
Poland
lukasz.balbus@pwr.wroc.pl,
[2]*Arizona State University,*
United States
skevin.reffett@asu.edu,
[3]*Warsaw Scool of Economics*,
Poland
[3]*lwozny@sgh.waw.pl*

Keywords*: Economy, Markov Perfect Equilibrium, Multigenerational altruistic model*

Abstract: *We consider an infnite horizon dynastic production economy with capital, labor. The economy is populated by dynasties consisting of a sequence of identical generations, each living one period, each caring about its own and successor generation. Sizes of each generation are equal and normalized to unity. The main results of the paper establish conditions sufficient for existence and uniqueness of measurable Markov perfect Nash equilibria. The results are constructive, i.e. we provide two approximation results allowing to construct the bounds of Markov Perfect Nash Equlibria set or to calculate the unique Markov Perfect Nash Equlibria. Moreover, our results emphasize both order-theoretic and geometrical properties of nonlinear fixed points operators, hence can be used to build globally stable asymptotically uniformly consistent numerical schemes for approximate solutions via Piccard iterations on approximate versions of our operators. Our results provide a new catalog of tools for the rigorous analysis (both theoretical and numerical) of Stationary Markov equilibrium for overlapping generations with stochastic production without commitement..*

In this paper we analyze equilibria in an infinite horizon stochastic production overlapping generations (OLG henceforth) economy with capital and labor and without commitment. Specifically from the game theoretical perspective we study a large class of stochastic intergererational paternalistic altruism games without commitment. In its deterministic incarnation, the model we consider dates back to the work of Phelps and Pollak [20], and consists of a sequence of identical generations, each living one period and deriving utility from its own consumption and leisure, as well as the consumption and leisure of its successor generation. The agents in the environment face a time-consistency problem between generations. In particular, each current generation has an incentive to deviate from a given sequence of bequests, consume a disportionate amount of current bequests, leaving little (or nothing) for subsequent generations.

Although existence and uniqueness of Markov perfect Nash equilibria (MPNE henceforth) in such models with inelastic labor supply is well known for both deterministic (see Leininger [15], Kohlberg [12]) and stochastic setting (papers of Amir [2], Nowak [17], or Balbus, Reffet and Wony [5]), surprisingly no existence and characterization are available for the case of elastic labor supply. In response to this predicament in this paper we establish conditions sufficient for existence of continuous, monotone MPNE as well as conditions guaranteeing uniqueness of MPNE in the space of a Borel measurable strategies. Finally we show conditions for existence of a unique corresponding invariant distribution on both the bounded or unbounded state space case. Let us mention that results are constructive, i.e. we provide two approximation results allowing to construct the bounds of MPNE set or to calculate the unique MPNE. Finally with a series of examples we show application and limitations of our results.

These results are important for a numerous reasons. Firstly they provide a rigorous set of tools allowing for a applied study of OLG economies with only partial commitment. Secondly the uniqueness and approximation results allow for a rigorous numerical characterization (including obtaining uniform error bounds) of MPNE in this class of economies. And thirdly the methods developed in this paper are general and hence can shed some new light on linked economic (e.g. dynamic economies with time consistency issues of Phelan and Stacchetti, Atkeson [19], studies on hyperbolic discounting Peleg and Yaari or Kusell and Smith [18,13]) or game theoretical problems (e.g. stochastic discounted supermodular games considered by Curtat [8], game of capital accumulation considered by Amir or resource extraction of Levhari and Mirman [16]) and others.

Specifically from the technical perspective our study of MPNE reduces to the analysis of fixed points for a appropriately defined operator on the strategy space. Hence we build on the decreasing operators methods applied first by Balbus, Reffett and Wozny [5] to a similar partial commitment economies with inelastic labor supply into a setting with two-dimensional choice variable space. This can be hence seen as a contribution to the literature of monotone operators similar to the one proposed (in case of elastic labor supply) by such authors as Coleman and Datta, Mirman and Reffett ([7] and [9]). Similarly to previous paper of Balbus, Reffett and Wozny [5] the fixed point theorems used here to show uniqueness are base on geometrical properties of monotone mappings defined in abstract cones found in the works of Guo and Lakshmikantham or Guo, Cho and Zhu([11] or [10]).

Our analysis of an optimization and fixed point problem with two choice variables for each generation interestingly stresses the discrepancies between methods used here or in Balbus, Reffett and Wozny [5] as opposed to the ones applied by Amir or Nowak ([2] or [17]). Specifically the operator used in our proof of uniqueness theorem is defined on the set of bounded functions on the states spaces and assigns for any expected utility of the next generation its best response expected utility of a current generation. This operator is hence a value function operator defined on the space of functions, whose construction is motivated by Abreu, Pearce and Stacchetti [1] (APS henceforth) operator that could be defined (in this example) on the set of subsets of the value functions. This is a striking difference to the ‘‘direct methods’’ applied by Amir or Nowak ([2] or [17]). As a result, on the one hand, methods we develop in this paper can be seen as a generalization to a two dimensional setting of an ‘‘inverse procedure’’ linking choice variables with their values proposed by Coleman [6]. But on the other, and more important, hand our contribution can be seen as a consistent way of sharpening the equilibrium characterization results obtained by Kydland and Prescott [14] or APS type methods for the study of consistency problems. Specifically when applying APS techniques, although existence arguments can be addressed under very general conditions, the rigorous characterization of the set of dynamic equilibrium policies (either theoretically or numerically) is typically weak. Further, it has not yet been shown how to apply APS to obtain any characterization of the long-run stochastic properties of stochastic games (i.e., equilibrium invariant distributions and/or ergodic distributions). Once again value function methods proposed in this paper should be seen as a way of circumventing the mentioned APS predicaments.

References

[1] Abreu, D., D. Pearce, and E. Stacchetti (1990): Toward a theory of discounted repeated games with imperfect monitoring, Econometrica, 58(5), 1041-1063.

[2] Amir, R. (1996): Strategic intergenerational bequests with stochastic con- vex production, Economic Theory, 8, 367-376.

[3] Amir, R. (2002): Discounted stochastic games, Annals of Operations Research, 114, 39-56.

[4] Atkeson, A. (1991): International Lending with Moral Hazard and Risk of Repudiation, Econometrica, 59(4), 1069-1089.

[5] Balbus, L., K. Reffett, and L. Wony (2009): A Constructive Geo- metrical Approach to the Uniqueness of Markov Perfect Equilibrium in Stochastic Games of Intergenerational Altruism, Arizona State University.
[6] Coleman, Wilbur John, I. (2000): Uniqueness of an Equilibrium in Infinite-Horizon Economies Subject to Taxes and Externalities, Journal of Economic Theory, 95, 71-78.
[7] Coleman, W. I. (1997): Equilibria in Distorted Infinite-Horizon Economies with Capital and Labor, Journal of Economic Theory, 72(2), 446-461.
[8] Curtat, L. (1996): Markov equilibria of stochastic games with comple- mentarities, Games and Economic Behavior, 17, 177-199.
[9] Datta, M., L. J. Mirman, and K. L. Reffett (2002): Existence and Uniqueness of Equilibrium in Distorted Dynamic Economies with Capital and Labor, Journal of Economic Theory, 103(2), 377-410.
[10] Guo, D., Y. J. Cho, and J. Zhu (2004): Partial ordering methods in nonlinear problems. Nova Science Publishers, Inc., New York.
[11] Guo, D., and V. Lakshmikantham (1988): Nonlinear problems in abstract cones. Academic Press, Inc., San Diego.
[12] Kohlberg, E. (1976): A model of economic growth with altruism between generations, Journal of Economic Theory, 13, 1-13.
[13] Krusell, P., and A. Smith (2003): ConsumptionSavings Decisions with QuasiGeometric Discounting, Econometrica, 71(1), 365-375.
[14] Kydland, F., and E. Prescott (1980): Dynamic optimal taxation, ra- tional expectations and optimal control, Journal of Economic Dynamics and Control, 2(1), 79-91.
[15] Leininger, W. (1986): The existence of perfect equilibria in model of growth with altruism between generations, Review of Economic Studies, 53(3), 349-368.
[16] Levhari, D., and L. Mirman (1980): The great fish war: an example us- ing a dynamic Cournot-Nash solution, Bell Journal of Economics, 11(1), 322-334.
[17] Nowak, A. S. (2006): On perfect equilibria in stochastic models of growth with intergenerational altruism, Economic Theory, 28, 73-83.
[18] Peleg, B., and M. E. Yaari (1973): On the Existence of a Consistent Course of Action when Tastes are Changing, Review of Economic Studies, 40(3), 391-401.
[19] Phelan, C., and E. Stacchetti (2001): Sequential equilibria in a Ram- sey tax model, Econometrica, 69(6), 1491-1518.
[20] Phelps, E., and R. Pollak (1968): On second best national savings and game equilibrium growth, Review of Economic Studies, 35, 195-199.

Clearing Supply and Demand under Bilateral Constraints*

Olivier Bochet[1], **Rahmi Ilkilic**[2] and **Herve Moulin**[3]

[1]*University of Bern and Maastricht University;*
olivier.bochet@vwi.unibe.ch
[2]*Maastricht University;*
r.ilkilic@algec.unimaas.nl
[3]*Rice University;*
moulin@rice.edu

Keywords: *Bipartite graph, bilateral trade, Strategy-proofness, Equal treatment of equals, Single-peaked preferences*

In a moneyless market, a non storable, non transferable homogeneous commodity is reallocated between agents with single-peaked preferences. Agents are either suppliers or demanders. Transfers between a supplier and a demander are feasible only if they are *linked*, and the links form an arbitrary bipartite graph. Typically, supply is short in one segment of the market, while demand is short in another.

Information about individual preferences is private, and so is information about feasible links: an agent may unilaterally close one of her links if it is in her interest to do so.

Our *egalitarian transfer* solution rations only the long side in each market segment, equalizing the net transfers of rationed agents as much as permitted by the bilateral constraints. It elicits a truthful report of both preferences and links: removing a feasible link is never profitable to either one of its two agents. Together with efficiency, and a version of equal treatment of equals, these properties are characteristic.

* This paper benefited from the comments of seminar participants at CORE, the universities of Edinburgh, St Andrews, Venice, Warwick, Pompeu Fabra, and the London Business School. Special thanks to Anna Bogomolnaia, Bettina Klaus, Jeremy Laurent-Lucchetti, Karl Schlag and Jay Sethuraman for stimulating discussions. Rahmi Ilkilic acknowledges the support of the European Community via Marie Curie Grant PIEF-GA-2008-220181. Moulin's research was supported by MOVE at the Universitat Autonoma de Barcelona.

Product Diversity under Monopolistic Competition*

Igor A. Bykadorov

Sobolev Institute of Mathematics SB RAS
Russia
bykad@math.nsc.ru

Keywords: *Monopolistic competition, Retailing, Pricing, Product diversity*

This paper analyzes the shift from competitive retailing to a dominant retailer in a standard industrial organization model of monopolistically competitive manufacturing. The model approximates the changing conditions in retailing markets in contemporary Russia and other developing markets of the FSU.

In 2000-s, Russia and other developing markets of FSU have shown dramatic growth of chain-stores and similar retailing firms. Inspired by Wal-Mart and other successful giants abroad, Russian food traders like Perekrestok and Pyaterochka has gained considerable shares of the market and noticeable market power, both in Moscow and in province. This shift in the market organization was suspected by newspapers for negative welfare effects, for upward pressure on prices and inflation. Public interest to this question is highlighted by anti-chain-stores bill currently debated in Russian Parliament (Duma).

Leaving aside the empirical side of this question, this paper focuses on constructing and analyzing the adequate model of vertical market interaction, suitable for Russian retailing markets of food, clothes and durables. We step aside from the traditional models of monopolistic or oligopolistic vertical interaction or franchising, and stick instead to more modern monopolistic-competition model of an industry in the

* Supported in part by RHF (grant no. 09-02-00337).

The work on this paper was supported by an individual grant № R08-1071 from the Economics Education and Research Consortium, Inc. (EERC), with funds provided by the Eurasia Foundation (with funding from the US Agency for International Development), The World Bank Institution, the Global Development Network and the Government of Sweden.

Dixit-Stiglitz [3] spirit, but combined with vertical interaction (this combination is rather new, being pioneered by Chen [1] and Hamilton-Richards [4]).

Our departure from the Chen- Hamilton-Richards approach is that it is the retailer who is exercising the monopsony power, while the production is organized as monopolistic-competition industry with free entry. This hypothesis seems rather realistic, at least for developing markets. Indeed, there are numerous evidences in economic newspapers that each of several big retailers has much stronger bargaining power than quite numerous manufacturers and importers of consumer goods like sausages, shirts, etc. (even such big international companies as Coca-Cola are not strong enough to enforce their terms of trade to Russian retailers).

Therefore our stylized model of market concentration considers a monopsonistic /monopolistic retailer (as a proxy of an oligopsonistic/oligopolistic retailer) dealing with a continuum [0, N] of Dixit-Stiglitz manufacturers and a representative consumer. The consumer's labor is supplied to the market inelastically, being the only production factor. There are two types of goods. The first "commodity" consists of many varieties, for instance, milk of different brands, and the number of firms/varieties is determined endogeneously by the free entry or similarly, taking into account the fixed and variable costs of manufacturers. The second commodity is the numeraire representing other (perfectly competitive) goods. The income effect is neglected. "Very many" similar manufacturers are playing a sequential game with one retailer. Each manufacturer has a fixed cost and a variable cost, he produces a single variety of the "commodity" and sets the price for this variety. The most natural timing of the model is when the retailer starts with announcing her markup policy correctly anticipating the subsequent manufacturers' responses, and simultaneously chooses the scope of varieties/firms to buy from. Then the manufacturers come up with their prices and the market clears the quantities. Both sides take into account the demand profile generated by a consumer's quasi-linear utility function. This organization of the industry is compared to the pre-concentration situation, modeled as leading manufacturers. This type of interaction is similar in some sense with the concept of "common agency" considered in contract theory. Another model version concerns the myopic (Nash) behavior of both sides being unable to predict and influence the market.

The questions addressed are: Does the emergence of the monopsonistic retailer enhance or deteriorate welfare, and how much? Which retailer is worse: the myopic or the wise one? What should be the guidelines for public regulation (if any) in this area?

In the special case of quadratic valuations (linear demands, see Section 3.2 of the book P.-P. Combes, T. Mayer and J.-F. Thisse [2]) it turns out that generally the concentration enhances social welfare. This effect, unexpected for non-economists, actually is not too surprising, because it reminds the switch from a two-tier monopoly to a simple monopoly. The welfare increase here, however, can go not only through the growth of the total quantity, but in some cases through growing total profits and, notably, through decreasing excessive diversity. The retailer is shown to be interested in restricting the diversity, but it is not harmful itself.

In the case of the governmental regulation of the monopsonistic retailer through Pigouvian taxes, it turns out that subsidies instead of taxes are needed, that also reminds similar effect known for a simple monopoly/monopsony.

A possible extension is to find similar result under another kind of utility: CES function. Another extension is to find, whether the modern practice of the entrance fee levied on the manufacturers by the retailers brings welfare gains or losses.

References

[1] Chen Z. Monopoly and Production Diversity: The role of retailer Countervailing Power. - November 8, 2004, 25 p. – Discussion Paper of Carleton University.

[2] Combes P.-P., Mayer T. and Thisse J.-F. Economic Geography. Princeton University Press, 2008.

[3] Dixit A.K. and Stiglitz J.E. Monopolistic competition and optimum product diversity. American Economic Review, 67 (1977), p. 297-308.

[4] Hamilton S.F. and Richards T.J. Comparative Statics for Supermarket Oligopoly with Applications to Sales Taxes and Slotting Allowances. - May 31 2007, 22 p. Selected Paper prepared for presentation at the American Agricultural Economics Association Annual Meeting, Portland, OR, July 29 - August 1, 2007.

Manufacturer-Retailer-Consumer Relationship in the Differential Games Framework

Igor A. Bykadorov[1], **Andrea Ellero**[2] and **Elena Moretti**[3]

[1]*Sobolev Institute of Mathematics SB RAS*
Russia
bykad@math.nsc.ru

[2,3]*Università Ca'Foscari Di Venezia, Venezia*
Italy
ellero@unive.it
emoretti@unive.it

Keywords: *Vertical channel, Trade discount, Sales motivation*

The goal of this paper is to model the effects of wholesale price and retail price controls on manufacturer's and retailer's profits, taking explicitly into account the retailer's sales motivation and performance. We consider a stylized distribution channel where a manufacturer sells a single kind of good to a retailer and consider the manufacturer-retailer relationship is modeled as a differential game.

To earn a reasonable profit the members of a distribution channel often adopt rather simple pricing techniques. For example, manufacturers may use cost-plus pricing, defining the price by simply adding a desired profit margin to the marginal cost. In a similar fashion, retailers very often determine shelf prices adding a fixed percentage markup to the wholesale price.

The main advantage of simple policies is that they are...easy to be applied. But this blind approach to pricing does not provide tools to manufacturers in order to encourage retailers to sell and retailers, in turn, cannot adequately stimulate consumer to buy.

In this paper we consider a vertical control distribution channel where a manufacturer sells a single kind of good to a retailer. We will focus on the effects of trade promotions, a widely used dynamic pricing strategy that manufacturers can exploit to raise sales. With trade promotions an incentive mechanism is used to drive other channel members' behaviors.

In particular we investigate the relationships between the members of a distribution channel by means of optimal control models in a stylized vertical distribution channel: a manufacturer serves a single segment market through a single retailer and a contract fixes a trade discount policy which will be followed by the contractors.

We assume that a discount in wholesale price increases the retailer's sales motivation and consequently it increases sales. We study both the manufacturer's and retailer's profit maximization problems as optimal control models. Our starting point is given by a couple of models presented in [1] and [2]. In those models the manufacturer sells to a single product to a single retailer: her aim is to maximize the total profit in a given time period. The manufacturer controls the discount on wholesale price (trade discount) allowed to the retailer, this way she can raise her sales because the retailer transfers a part of the discount to the shelf price (pass-through). Moreover, if the retailer controls the amount of trade discount transferred by the to the market price (pass-through) he can decide to keep part of the incentive for himself and will be therefore more motivated in selling the specific product, thus giving another upward push to sales. The optimal control of manufacturer's and retailer's profit via trade discount and pass-through is embedded in a differential game framework. We consider in particular the case when feasible controls of the manufacturer and the retailer are constant and/or piece-wise constant. Some preliminary results can be found in [3].

References

[1] Bykadorov I., Ellero A., Moretti E. Optimal control of trade discount in a vertical distribution channel – Rendiconti per gli Studi Economici Quantitativi, 2005, vol.15 (35), no.1, p.121-129.

[2] Bykadorov I., Ellero A., Moretti E., Vianello S. The role of retailer’s performance in optimal wholesale price discount policies – European Journal of Operational Research, 2009, vol. 194, no. 2, p. 538–550.

[3] Bykadorov I.A., Ellero A., Moretti E. Trade discount policies in the differential games framework: preliminary results – Venezia (Italia), Università Ca’Foscari di Venezia, Dipartimento di Matematica Applicata, 2005, report n.130/2005, 27 p.

Bargaining One-dimensional Policies and the Efficiency of Super Majority Rules

Daniel Cardona[1] and **Clara Ponsati**[2]

[1]*Universitat de les Illes Balears,*
[2]*Institut d'Analisi Economica (CSIC),*
Barcelona

We consider negotiations selecting one-dimensional policies. Individuals have single-peaked preferences, and they are impatient. Decisions arise from a bargaining game with random proposers and (super) majority approval, ranging from the simple majority up to unanimity. The existence and uniqueness of stationary sub-game perfect equilibrium is established, and its explicit characterization provided. We supply an explicit formula to determine the unique alternative that prevails, as impatience vanishes, for each majority. As an application, we examine the efficiency of majority rules. For symmetric distributions of peaks unanimity is the unanimously preferred majority rule. For asymmetric populations rules maximizing social surplus are characterized.

Journals in Game Theory

GAMES AND ECONOMIC BEHAVIOR

Editor
Ehud Kalai

ELSVIER

Dividing and Discarding: A Procedure for Taking Decisions with Non-Transferable Utility

Vinicius Carrasco[1] and **William Fuchs**[2]

[1]*PUC-Rio,*
Brazil
vncarrasco@gmail.com
[2]*University of California, Berkeley,*
United States
wfuchs@haas.berkeley.edu

Keywords: *Decision-Making with NTU, Asymmetric Information, Bayesian Implementation, Communication*

We consider a setting in which two players must take a single action. The analysis is done within a private values model in which (i) the players' preferences over actions are private information, (ii) utility is non-transferable, (iii) implementation is Bayesian and (iv) the welfare criterion is utilitarian. We characterize an optimal allocation rule. Instead of asking the agents to directly report their types, this allocation can be implemented dynamically. The agents are asked if they are to the left or to the right of a given cutoff, if both reports agree, the section of the interval which none preferred is discarded and the process continues until one agent chooses left and the other right. In that case, this last cutoff is implemented. When types are uniformly distributed, this implementation can be carried out by a Principal who lacks commitment, implying this process is an optimal communication protocol.

Extension of a Game Problem in the Class of Finitely Additive Measures

Alexander G. Chentsov[1] and **Julia V. Shapar**[2]

[1,2]*Institute of Mathematics and Mechanics,*
Ural Branch of Russian Academy of Sci.,
Ekaterinburg, Russia
[1]*e-mail: chentsov@ imm.uran.ru*
[2]*shaparuv@mail.ru*

We consider the game problem

$$\Phi(u,v) \triangleq f_0\left(\left(\int_{I_1}\alpha_i u d\eta_1\right)_{i\in\overline{1,k}}, \left(\int_{I_2}\beta_j v d\eta_2\right)_{j\in\overline{1,l}}\right) \to \sup_v \inf_u, \tag{1}$$

where $u \in U$ and $v \in V$. In addition, U and V are nonempty sets of real-valued functions on I_1 and I_2 respectively $(I_1 \neq \varnothing, I_2 \neq \varnothing)$. Moreover, the concrete choice of u and v should be satisfy to the constraints

$$\left(\int_{I_1}\gamma_i u d\eta_1\right)_{i\in\overline{1,p}} \in Y,\ \left(\int_{I_2}\omega_j v d\eta_2\right)_{j\in\overline{1,q}} \in Z\ , \tag{2}$$

where Y and Z are nonempty closed sets in $\mathbb{R}^p$ and $\mathbb{R}^q$ respectively. We suppose that η_1 and η_2 are nonnegative real-valued finitely additive measures (FAM) defined on the semialgebras L_1 and L_2 of subsets I_1 and I_2 respectively. So,

$$(I_1, L_1, \eta_1), (I_2, L_2, \eta_2) \tag{3}$$

are finitely additive spaces with a measures.

In particular, in (3) the case of standard measure spaces in assumed. We suppose that f_0 is a continuous function on $\mathbb{R}^k \times \mathbb{R}^l$, U and V are nonempty subsets of the manifolds of step-functions on I_1 and I_2 respectively, and, moreover, $\alpha_i, \beta_j, \gamma_i, \omega_j$ are uniform limits of the corresponding sequences of step-functions. We suppose that for some $a \in]0,\infty[$ and $b \in]0,\infty[$

$$\left(\int_{I_1}|u| d\eta_1 \leq a\ \forall u \in U\right) \,\&\, \left(\int_{I_2}|v|\, d\eta_2 \leq b\ \forall v \in V\right). \tag{4}$$

We investigate the influence (on the result) of the constraint weakening in (2). Namely, we consider the replacements

$$Y \to Y^{\varepsilon},\, Z \to Z^{\delta}\,, \tag{5}$$

where $\varepsilon > 0$ and $\delta > 0$. The limit relations under $\varepsilon \downarrow 0$ and $\delta \downarrow 0$ are investigated (the problem (1) under constraints (2) can be unstable; see example in [1]). The representation of asymptotics of maximin (1) under constraints with the replacements (5) is established. In this representation, the extension construction in the class of FAM is used. For this we consider the spaces $A(L_1)$ and $A(L_2)$ of real-valued FAM of bounder variation defined on L_1 and L_2 respectively. Under constructing the generalized problems the replacements

$$(u,v) \to (\mu,\nu),$$

where μ and ν are FAM, is used. Here $\mu \in \tilde{U}$ and $\nu \in \tilde{V}$ where $\tilde{U}$ and $\tilde{V}$ are realized as $*$-weak closures of U and V under the immersion in $A(L)$ by indefinite integrals with respect to η_1 and η_2 respectively: $u \to u * \eta_1,\ v \to v * \eta_2$. In this connection, see [2]. By (4) the sets $\tilde{U}$ and $\tilde{V}$ are nonempty
$*$-weak compactums. We replace (2) by conditions

$$\left(\int_{I_1} \gamma_i d\mu\right)_{i\in\overline{1,p}} \in Y,\ \left(\int_{I_2} \omega_j d\nu\right)_{j\in\overline{1,q}} \in Z\,. \tag{6}$$

By analogy with (1) is replaced by the generalized game problem

$$\tilde{\Phi}(\mu,\nu) \triangleq f_0\left(\left(\int_{I_1} \alpha_i u d\mu\right)_{i\in\overline{1,k}}, \left(\int_{I_2} \beta_j d\nu\right)_{j\in\overline{1,q}}\right) \to \max_{\nu}\min_{\mu} \tag{7}$$

in addition, $\mu \in \tilde{U}$ and $\nu \in \tilde{V}$ satisfy to (6). Then maximin of (7) fully defines asymptotics of realized maximins under weakening constraints (see (5)). We note that the maximin (7) can be differ from usual maximin under constraints (2). But, under some additional conditions the coincidence of usual and generalized maximins takes place. In the following, we suppose that along with the equalities

$$\tilde{U} = \mathrm{cl}(\{u * \eta_1 : u \in U\}, \tau_*(L_1)),\quad \tilde{V} = \mathrm{cl}(\{v * \eta_2 : v \in V\}), \tau_*(L_2),$$

where $\tau_*(L_1)$ and $\tau_*(L_2)$ are the $*$-weak topologies of $A(L_1)$ and $A(L_2)$ the equalities

$$\tilde{U} = \mathrm{cl}(\{u * \eta_1 : u \in U\}, \tau_0(L_1)),\quad \tilde{V} = \mathrm{cl}(\{v * \eta_2 : v \in U\}, \tau_0(L_2)) \tag{8}$$

are required; here $\tau_0(L_i)$ is topology of $A(L_i)$ defined in [3, (4,2,9)] ($\tau_0(L_i)$ is topology of the Tichonoff degree subspace of real line in the discrete topology for which L_i is used as the index set). We note that the conditions (8) are valid in many practically interesting cases. Now, we give the following examples (we denote by $B_0(I_j, L_j)$ (by $B_0^+(I_j, L_j)$) the sets of all step-functions (all nonnegative step-functions) on I_j in the sense of (I_j, L_j)):

$1')$ $U = \left\{ u \in B_0(I_1, L_1) \mid \int_{I_1} |u| d\eta_1 \leq a \right\},$

$2')$ $U = \left\{ u \in B_0^+(I_1, L_1) \mid \int_{I_1} u d\eta_1 \leq a \right\},$

$3')$ $U = \left\{ u \in B_0^+(I_1, L_1) \mid \int_{I_1} u d\eta_1 = a \right\},$

$4')$ if $r \in \mathbb{N},\ (L_i)_{i \in \overline{1,r}} : \overline{1,r} \to L,\ (c_i)_{i \in \overline{1,r}} : \overline{1,r} \to [0, \infty[$ and $I_1 = \bigcup_{i=1}^{r} L_i,$ then we can be suppose that $U = \left\{ u \in B_0(I_1, L_1) \mid \int_{L_k} |u| d\eta_1 \leq c_k\ \forall k \in \overline{1,r} \right\};$

$1'')$ $V = \left\{ v \in B_0(I_2, L_2) \mid \int_{I_2} |v| d\eta_2 \leq b \right\},$

$2'')$ $V = \left\{ v \in B_0^+(I_2, L_2) \mid \int_{I_2} v d\eta_2 \leq b \right\},$

$3'')$ $V = \left\{ v \in B_0^+(I_2, L_2) \mid \int_{I_2} v d\eta_2 = b \right\},$

$4'')$ if $s \in \mathbb{N},\ (L_i)_{i \in \overline{1,s}} : \overline{1,s} \to L,\ (\tilde{c}_i)_{i \in \overline{1,s}} : \overline{1,s} \to [0, \infty[$ and $I_2 = \bigcup_{i=1}^{s} L_i,$ then we can be suppose, that $V = \left\{ v \in B_0(I_2, L_2) \mid \int_{L_k} |v| d\eta_2 \leq \tilde{c}_k\ \forall k \in \overline{1,s} \right\}.$

In addition, we can be use $1')$ - $4')$ and $1'')$ - $4'')$ in arbitrary combinations: for example the variant $1'), 3'')$ is possible. Moreover, we suppose that γ_i and ω_j are step-functions. Namely, let

$$\left(\gamma_i \in B_0(I_1, L_1)\ \ \forall i \in \overline{1,p} \right) \& \left(\omega_j \in B_0(I_2, L_2)\ \ \forall j \in \overline{1,q} \right). \tag{9}$$

Under the above-mentioned additional conditions (8) and (9) and under the compatibility of (2), the usual and generalized maximins coincidence: we have the stability by maximins.

References

[1] Chentsov A. G., Shapar Ju. V. About one the game problem with the approximate realization to constraints.// Dokladi Akademii Nauk. – 2009. –- V. 427, 2 –-P.170 - 175.

[2] Chentsov A. G. Elements of finite-additive measure theory, *I*.. - Ekaterinburg: USTU-UPI, 2008. - 388 p. (Russian).

[3] Chentsov A. G. Finitely additive measures and relaxations of extremal problems. – New York, London and Moscow: Plenum Publishing Corporation, 1996. –- 244 p.

NE-Strong Cooperative Solutions in Differential Games

Sergey Chistyakov[1] and **Leon Petrosyan**[2]

[1,2]*St Petersburg University*
Russia
[1]*svch50@mail.ru*
[2]*formail@list.ru*

Keywords: *Strong Nash equilibrium, Cooperative trajectory, Imputation distribution procedure, Differential game*

The problem of strategically strongly provided cooperation in n-persons differential games with integral payoffs is considered. Based on initial differential game the new associated differential game (CD-game) is designed. In addition to the initial game it models the players actions connected with transition from the strategic form of the game to cooperative with in advance chosen principle of optimality. The model provides possibility of refusal from cooperation at any time instant t for each player. As cooperative principle of optimality imputation from the core selected in appropriate manner is considered. In the bases of CD-game construction lies the so-called imputation distribution procedure described earlier in [1] (see also [2]). The theorem established by authors says that if at each instant of time along the conditionally optimal (cooperative) trajectory the future payments to each coalition of players according to the imputation distribution procedure exceed the maximal guaranteed value which this coalition can achieve in CD-game, then for every $e > 0$ there exist a strong Nash equilibrium in the class of recursive strategies first introduced in [3]. In other words, the mentioned equilibrium exists, if in any subgame along the conditionally optimal trajectory the core is not void. The proof of this theorem uses results and methods published in [3], [4].

References

[1] Petrosjan L.A., Danilov N.N. Stability of Solutions in nonzero-sum Differential Games with Integral Payoffs, Viestnik Leningrad University, 1979, N1, pp.52-59.
[2] Petrosjan L.A. The Shapley Value for Differential Games, Annals of the International Society of Dynamic Games, Vol.3, Geert Jan Olsder Editor, Birkhauser, 1995, pp. 409-417.

[3] Chistyakov S.V. O beskoalizionnikx differenzial'nikx igrakx (On Noncooperative Differential Games) Dokl. Akad. Nauk, 1981, vol. 259, no. 5, pp. 1052-1055; English transl. in Soviet Math. Dokl. 24 (1981), no. 1, pp.166-169.
[4] Chistyakov S.V. Value Operators for zero-sum Games, St. Petersburg: St. Petersburg Univ. Press, 1999, p.62.

Multipath Multiuser Scheduling Game for Elastic Traffic

Julia Chuyko [1], **Tatiana Polishchuk**[2], **Vladimir Mazalov** [3] and **Andrei Gurtov** [4]

[1,3]*Institute of Applied Mathematical Research,*
Karelian Research Center of RAS
185910, Russia, Petrozavodsk, Pushkinskaya str., 11
phone: (8142)781108, fax: (8142)766313,
URL: http://mathem.krc.karelia.ru
[2,4]*HIIT, Aalto University,*
Finland
[1]*julia@krc.karelia.ru*
[2]*tatiana.polishchuk@hiit.fi*
[3]*vmazalov@krc.karelia.ru*
[4]*gurtov@hiit.fi*

Keywords*: Wardrop equilibrium, Selfish routing, Traffic delay, Social costs*

Abstract: *Our multipath multiuser routing optimization problem is based on Wardrop model of splittable traffic routing. Minimization of the end-to-end traffic delay for each user is the criterion of optimality. We consider a routing game with traffic delay functions* $1-e^{-\alpha_{ie}\delta_e}$. *Wardrop Equilibria and their properties in this model are objects of the research.*

Our multipath multiuser routing optimization problem is based on Wardrop model [1, 2, 3] of splittable traffic routing. Minimization of the end-to-end traffic delay for each user is the criterion of optimality.

The problem is considered as the game $\Gamma = \langle n, m, w, f \rangle$, where n users send their traffic through m parallel routes from the source s to destination t. Each user i wants to send traffic of the amount w_i from s to t. Each path e has a characteristic $\alpha_{ie} > 0$.

Users act selfish and choose routes to minimize their maximal traffic delay. They can split their traffic and send it on several or all paths simultaneously. User's i strategy is $x_i = \{x_{ie} \geq 0\}$, where x_{ie} is the traffic amount that he sends on the path e so that $\sum_{e=1}^{m} x_{ie} = w_i$. Then $x = (x_1, \ldots, x_n)$ is users strategy profile. Denote for the original profile x the new profile $(x_{-i}, x_i') = (x_1, \ldots, x_{i-1}, x_i', x_{i+1}, \ldots, x_n)$ where the user i changes his strategy from x_i to x_i' and all other users keep their strategies the same as in x.

The load of the path e is a function $\delta_e(x)$ that is continuous and non-decreasing by x_{ie}. A continuous traffic delay function $f_{ie}(x) = f_{ie}(\delta_e(x))$ is defined for each user i and each route e. It is non-decreasing by the path load and hence by x_{ie}.

Function $PC_i(x)$ defines an individual i-th user's costs. Each user i tries to minimize his individual costs – the maximal traffic delay among the routes that he uses

$$PC_i(x) = \max_{e:x_{ie}>0} f_{ie}(x).$$

A strategy profile x is a Wardrop equilibrium iff for each i holds: $if \quad x_{ie} > 0, f_{ie}(x) = \min_l f_{il}(x) = \lambda_i \quad if \quad x_{ie} = 0,\ f_{ie}(x) \geq \lambda_i.$

Social costs are the total costs of the system as a result of using parallel routes of the network:

$$SC(x) = \sum_{i=1}^{n}\sum_{e=1}^{m} x_{ie} f_{ie}(x).$$

A social optimum is a solution of a minimization problem $SC(x) \to \min_{x \text{ is a strategy profile}}$. Price of Anarchy is a ratio of equilibrium social costs in the worst case equilibrium and optimal social costs.

$$PoA(\Gamma) = \max_{x \text{ is an equilibrium}} \frac{SC(x)}{SC_{opt}}.$$

In this work we consider a routing game with traffic delay functions $1 - e^{-\alpha_{ie}\delta_e}$ in case where for each path e its traffic delay is the same for each user. Experimental modeling confirms an adequacy of such delay function and explains a sence of parameters α. Wardrop Equilibria and their properties in this model are objects of the research. We obtain that a Wardrop equilibrium is any situation where loads are distributed by routes as follows:

$$\sum_{i=1}^{n} x_{ie} = \delta_e(x) = \frac{W}{\alpha_e \sum_{e=1}^{m} \frac{1}{\alpha_e}}, \forall e \in \{1,\ldots,m\},$$

and the equilibrium social costs are

$$SC(x) = W\Big(1 - e^{-\frac{W}{\sum_{e=1}^{m} \frac{1}{\alpha_e}}}\Big).$$

Also we prove, that the Price of Anarchy is about 1.3 for this model.

Our research is supported by TEKES as part of the Future Internet program of TIVIT (Finnish Strategic Centre for Science, Technology and Innovation in the field of ICT) and by Russian Foundation for Basic Research (grant N 10-01-00089-a).

References

[1] Gairing M., Monien B., Tiemann K. Routing (Un-) Splittable Flow in Games with Player-Specific Linear Latency Functions // Proceedings of the 33rd International Colloquium on Automata, Languages and Programming (ICALP 2006), LNCS 4051, 2006, 501 – 512.

[2] Wardrop J.G. Some theoretical Aspects of Road Traffic Research // Proceedings of the Institute of Civil Engineers, 1952, II, 1, 325 –278.

[3] Mazalov V., Monien B., Schoppmann F., Tiemann K. Wardrop Equilibria and Price of Stability for Bottleneck Games with Splittable Traffic // Proceedings of 2nd international Workshop on Internet & Network Economics (WINE 2006, Patras / Greece), 2006, to appear in Lecture Notes in Computer Science.

[4] Julia V. Chuiko, Vladimir V. Mazalov. Nash Equilibrium in Splittable Traffic Routing Problem // Proceedings of Kosen Workshop MTE2008 - Mathematics, Technology and Education - Ibaraki National College of Thechnology, Hitachinaka, Ibaraki, Japan. 2008. Pp. 13-18.

[5] A.Gurtov, T. Polishchuk, Secure Multipath Transport for Legacy Internet Applications // Proceedings of BROADNETS'09, Madrid, Spain, September 2009.

МАТЕМАТИЧЕСКАЯ ТЕОРИЯ ИГР И ЕЕ ПРИЛОЖЕНИЯ

Выходит ежеквартально с января 2009 года

Simultaneous Quantity and Price - Choice in a Sequential Movement Game (SQAP)

Daniel Cracau

Research Associate of Prof. Dr. Abdolkarim Sadrieh,
Chair in E-Business,
Faculty of Economics and Management,
University of Magdeburg
Germany
cracau@ovgu.de

Keywords: *Homogeneous duopoly, Price and quantity competition, Second mover advantage*

Abstract: *I investigate an oligopoly model with homogeneous goods, sequential actions and simultaneous price and quantity choice. In a duopoly game a pure-strategy noncooperative equilibrium with both firms making positive profits is derived. Furthermore it can be shown that when the number of firms converges towards infinity, the leader sets a price equal to marginal cost and is the sole firm serving the market.*

The simultaneous choice of prices and quantities has become relevant for research with Levitan/Shubik (1978). Ten years later Milton Friedman establishes that "there is no pure-strategy noncooperative equilibrium" in those games Friedman (1988). He calls the potentially existing mixed-strategy equilibria not plausible and therefore investigates two models with two firms choosing first quantities simultaneously and then prices simultaneously and vice versa.

Several models are developed dealing with the difficult question of the right strategic variable and the right timing of decisions. For instance, Tasnádi (2006) analyzes a mixed oligopoly with one firm choosing its price and the other firm choosing its quantity. In an extension of his model, the choice of the strategic variable itself is endogenized. Looking at a price choice as well as a capacity choice in a dynamic setting, Benoit/Krishna (1987) figure out that equilibrium outcomes depend on the flexibility of the capacity choice.

If goods are assumed to be heterogeneous, the research is often focussed on the question which type of competition is the best with regard to social welfare. Singh/Vives (1984) show that it is a dominant strategy for the firms to set quantities. Claiming that these results are sensitive to duopoly assumptions, Hackner (2000) points out that neither price nor quantity competition can be said to be more efficient in general. Finally, Hsu/Wang (2005) show that total surplus is always higher under price competition than under quantity competition.

Bárcena-Ruiz (2007) introduces a public firm maximizing social welfare in the competition with private firms. He shows that in this situation, firms prefer to set prices simultaneously in contrast to the classical competition with only private firms, where sequential price setting is prefered.

In my model, the firms have to choose quantity and price simultaneously but in an exogenous order. The derivation of that equilibrium is widely in line with the idea introduced by Levitan/Shubik (1978).

References

[1] Baird Douglas G., Gertner Robert H., Picker Randal C., *Game Theory and the Law,* Harvard University Press, Cambrideg Mass, 1994

[1] Bárcena-Ruiz, J. C. (2007): Endogenous Timing in a mixed Duopoly: Price Competition, Journal of Economics, Vol. 91, No. 3, pp. 263-272.

[2] Benoit, J. - P. / Krishna, V. (1987): Dynamic Duopoly: Prices and Quantities, The Review of Economic Studies, Vol. 54, No. 1, pp. 23-35.

[3] Boyer, M. / Moreaux, M. (1986): Perfect competition as the limit of a hierarchical market game, Economics Letters, Vol. 22, pp. 115-118. North Holland.

[4] Friedman, J. W. (1988): On the strategic importance of prices versus quantities, The RAND Journal of Economics, Vol. 19, No. 4, pp. 607-622.

[5] Hackner, J (2000): A Note on Price and Quantity Competition in Differentiated Oligopolies, Journal of Economic Theory, Vol. 93, pp. 233-239.

[6] Hsu, J. / Wang, X. H. (2005): On Welfare under Cournot and Bertrand Competition in Differentiated Oligopolies, Review of Industrial Organization, Vol. 27, pp. 185-191.

[7] Kreps, D. M. / Scheinkmann, J. A. (1983): Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes, The Bell Journal of Economics, Vol. 14, No.2, pp. 326-337.

[8] Levitan, R. / Shubik, M. (1978): Duopoly with Price and Quantity as Strategic Variables, Int. Journal of Game Theory, Vol. 7, Issue 1, pp. 1-11. Physica-Verlag, Vienna.

[9] Singh, N. / Vives, X. (1984): Price and Quantity Competition in a Differentiated Duopoly, RAND Journal of Economics, Vol. 15, pp. 546-554.

[10] Tásnadi, A. (2006): Price vs. quantity in oligopoly games, International Journal of Industrial Organization, 24, pp. 541-554.

Group Bargaining with Incomplete Information

Ekaterina Demidova[1] and **Pierfrancesco La Mura**[2]

[1,2]*HHL - Leipzig Graduate School of Management,*
Germany,
Russia
[1]*ekaterina.demidova@gmail.com*
[2]*plamura@gmail.com*

Keywords: *Bargaining, Incomplete information, Closed- open-doors*

Abstract: *In many real-life bargaining situations, in business life as well as in the political arena, the negotiating parties are typically not individual agents but rather groups of possibly heterogeneous individuals. In such situations, an agreement must reflect consensus not only between groups but also among members of the same group. We focus on the simplest of such scenarios, and extend Rubinstein's (1985) bilateral bargaining model with incomplete information to the case in which one of the negotiating parties is a couple. We investigate how the choice of negotiation format (specifically, open- vs. closed-door negotiations) affects the individual welfare of the participants, as well as the overall efficiency of the bargaining process.*

Situations in which a group of agents shares a joint entitlement to the outcome of a negotiation are not rare in everyday life, and include all those cases in which negotiations over public goods or resources are conducted through representatives, as typically happens in domestic or international political negotiations. In such cases, incomplete information and moral hazard often play a significant role.

Evidence from social psychology suggests that the strategic behavior of groups tends to differ significantly from that of individuals in comparable experimental settings. Yet, with a few notable exceptions, which are limited to finite-horizon settings (Putnam (1988), Perry and Samuelson (1994)), the theoretical literature on bargaining has mostly focused on the analysis of negotiations among individual agents.

As a first step towards the study of more general group bargaining situations we would like to investigate the simplest case of an infnite-horizon group bargaining problem: namely, the case of bilateral bargaining between an individual and a couple. Our starting point is Rubinstein's (1985) infinite-horizon, alternating-offers bargaining problem with one-sided incomplete information about time preferences. In our setting, player 1 is an individual who bargains against a couple, players b (he) and g (she). The couple shares a joint entitlement to the outcome of the negotiation: it is impossible for a member of the couple to strike an agreement with player 1, unless the other partner also agrees. While the types of players 1 and b are commonly known, player g can be either more patient or more impatient than her partner, and this is private knowledge of the couple. An offer is a proposal on the share of (unit) pie that player 1 should receive. In odd periods player 1 makes an offer, and b and g both reply with a Yes or No; in even periods b and g make (counter-) offers, and player 1 decides whether to accept the minimum of the two counteroffers (by replying Yes), or refuse both (by replying No). The game ends when either the current offer by player 1 is jointly accepted by the couple, or the minimum counteroffer by the couple is accepted by 1. For definiteness, we assume that the two members of the couple consume their share concurrently as a public good. Alternatively, one could assume that they further subdivide it between themselves through a subsequent non-cooperative negotiation.

With respect to what player 1 learns in the process of negotiations one can distinguish two representative scenarios. In the first scenario (open-door), player 1 learns both replies and both counteroffers. This case intuitively corresponds to situations in which the three players must all negotiate in the same room. In the second (closed-door), player 1 only learns the joint reply (whether both members of the couple agree, or not), and the minimum counteroffer. Intuitively, this is the case if the couple retires in a separate room whenever it is time to deliberate, and only transmits to player 1 a joint reply, or counteroffer, which has the approval of both partners. We show that even in such minimalistic setting the format of negotiations plays a significant role. In particular, we show that the choice of format (open- vs. closed-door) may have an impact on the efficiency of the outcome.

Our results imply that: (i) if the outcome in the open-door scenario is inefficient, it would also be inefficient when g bargains against 1 alone. Moreover, there exist cases in which a delay occurs when g bargains alone, but not when b and g bargain as a couple with open doors; and (ii) if the outcome with closed doors is inefficient, it would also be inefficient with open doors. Moreover, there exist cases in which a delay occurs with open doors, but not with closed doors. Hence, the closed-door scenario is always weakly more efficient than the open-door one, and in some cases strictly more efficient. The result supports the long-standing view that closed-door negotiations are generally more efficient than transparent ones.

References

[1] Baron, D. P., and J. A. Ferejohn, 1989. ‘‘Bargaining in Legislatures’’, American Political Science Review 83, 1181-1206.

[2] Bentham, J. 1816 [1999]. Political Tactics. Michael James, Cyprian Blamires, and Catherine Pease-Watkin (eds.) Oxford: Clarendon Press.

[3] Binmore, K.G. 1982. ‘‘Perfect Equilibria in Bargaining Models’’, International Center for Economics and Related Disciplines, London School of Economics, Discussion paper 82/58

[4] Bornstein, G., Kugler, T. and Ziegelmeyer, A. 2004. ‘‘Individual and Group Decisions in the Centipede Game: Are groups more rational players?’’ Journal of Experimental Social Psychology 40, p. 599-605.

[5] Bornstein, G., Kugler, T., Koche, M.G., Sutter, M. 2007. ‘‘Trust between Individuals and Groups: Groups are Less Trusting than Individuals but just as Trustworthy’’, Journal of Economic Psychology, Elsevier, vol. 28(6), p. 646-657.

[6] Chatterjee, K. and L. Samuelson (1990). ‘‘Perfect Equilibria in Simultaneous- Offers Bargaining’’, International Journal of Game Theory 19, 237-267.

[7] Fudenberg, D. and Tirole, J., 1996. Game Theory. The MIT Press, Massahusetts Institute of Technology.

[8] Hobbes, T. 1651 [1996]. Leviathan, Oxford: Oxford University Press.

[9] Holmstr ö m, B. 1979. ‘‘Moral Hazard and Observability’’, Bell Journal of Economics 10, p. 74-91.

[10] Keohane, R. 2002. Global Governance and Democratic Accountability. Unpublished manuscript, Duke University

[11] Krelle, W., (1975). ‘‘New Theory of Bargaining’’, Working Paper 70, Institut fr Gesellschafts- und Wirtschaftswissenschaften, Universität Bonn.

[12] Lewis, J., 1998. ‘‘Is the 'Hard Bargainingu' Image of the Council Misleading? The Committee of Permanent Representatives and the Local Elections Directive’’ Journal of Common Market Studies 36 (4): p. 479-504.

[13] Lewis, J., 2000. ‘‘The Methods of Community in EU Decision-Making and Administrative Rivalry in the Council Infrastructure. Journal of European Public Policy 7 (2) 261-89.

[14] Mill, J.S. 1861 [1972]. Considerations on Representative Government. London: Everymanu2019s Library, London.

[15] Muthoo, A., 1989. ‘‘A Note on the Strategic Role of Outside Options in Bilateral Bargaining’’, unpublished paper, Department of Economics, London School of Economics.

[16] Naurin, D. 2005. ‘‘Why Increasing Transparency in the European Union Will not Make Lobbyists Behave any Better than They Already Do?’’ paper for the EUSA Ninth Biennial International Conference Austin

[17] Perry, M. and Samuelson, L. 1994. ‘‘Open- versus Closed-Door Negotiations’’, The RAND Journal of Economics 25(2), p. 348-359.

[18] Perry, M. and P. J. Reny (1993), ‘‘A Non-cooperative Bargaining Model with Strategically Timed Offers’’, Journal of Economic Theory 59, 50-77.

[19] Polzer, J. 1996. ‘‘Intergroup Negotiations: The Effects of Negotiating Teams’’,Journal of Conflict Resolution 40 (4), p. 679-699.

[20] Prat A. 2003. ‘‘The Wrong Kind of Transparency’’, Unpublished manuscript, London School of Economics.

[21] Putnam, R., 1988. ‘‘Diplomacy and Domestic Politics: The Logic of Two-Level Games’’, International Organization 42 (3), pp. 427- 60.

[22] Risse, T., 2000. Let's Argue!: Communicative Action in World Politics. International Organization 54 (1):1-39.

[23] Roth, A.E. 1979. ‘‘Axiomatic Models of Bargaining, Berlin’’: Springer-Verlag.

[24] Rubinstein, A. 1982. ‘‘Perfect Equilibrium in a Bargaining Model’’Econometrica 50(1), 97-109.

[25] Rubinstein, A. 1985. ‘‘A Bargaining Model with Incomplete Information about Time Preferences’’, Econometrica 53(5), 1151-1173.
[26] Rubinstein, A. and Osborne, J.M., 1990. Bargaining and markets. Academic Press, San Diego.
[27] Schulte, E. 2007. Communication in Committees: Who should listen?
[28] Stasavage, D. 2004. ‘‘Open-Door or Closed-Door? Transparency in Domestic and International Bargaining’’, Unpublished working paper, London School of Economics, London.
[29] Stahl, D. O., II (1990). ‘‘Bargaining with durable offers and endogenous timing’’, Games and Economic Behavior 2, 173-187.
[30] Stahl, I. (1972), Bargaining Theory, Stockholm: Economics Research Institute, Stockholm School of Economics.
[31] Sutter, M. 2004. ‘‘Are Four Heads Better than Two? An experimental beauty-contest game with teams of different size’’, Economics Letters, Elsevier, vol. 88(1), p. 41-46.
[32] van Damme, E., R. Selten, and E. Winter (1990), ‘‘Alternating Bid Bargaining with a Smallest Money Unit’’, Games and Economic Behavior 2, 188-201.
[33] Wagner, R.H. 1984. ‘‘A Noncooperative Solution to a Two-Person Bargaining Game’’. unpublished paper.
[34] Watson, J. 1995 ‘‘Alternating-Offer Bargaining with Two-Sided Incomplete Information’’, Discussion paper 94-13R, University of California, San Diego.
[35] Wolinsky, A. (1987). ‘‘Matching, Search, and Bargaining’’, Journal of Economic Theory 42, 311-333.

Decomposition of Distributions over the Two-dimensional Integer Lattice and Multistage Bidding with Two Risky Assets

Victor Domansky and **Victoria Kreps**

St.Petersburg Institute for Economics and Mathematics
Russian Academy of Sciences
Russia
kreps@emi.nw.ru

Keywords: *Multistage bidding, Risky asset, Repeated game, Incomplete information, Optimal strategy*

In [1] we investigated the model of multistage bidding where r types of risky assets (shares) are traded between two stockmarket agents. The agents have different information on liquidation prices of traded assets. Before bidding starts the prices are determined by a chance move for the whole period of bidding according to a known to both players probability distribution over r-dimensional integer lattice. Player 1 is an insider. He knows the prices of all types of shares. Player 2 has no such information. Player 2 knows that Player 1 is an insider. At each step of bidding both players make simultaneously their bids, i.e. they post their prices for each type of shares. The player who posts the larger price for a share of given type buys one share of this type from his opponent for this price.

Any integer bids are admissible. Players aim to maximize the values of their final portfolios, i.e. money plus liquidation values of obtained shares. Both players are supposed to remember all previous bids. This allows Player 2 draw conclusions on real share prices from the actions of Player 1 and forces Player 1 to prevent such deduction. The described model was reduced to the zero-sum repeated games with lack of information on one side and with r-dimensional one-step actions with components corresponding to bids for various assets. In [1] it was established that

1) if expectations of share prices are finite, then the values of n-stage bidding games with assets of several types exist and does not exceed the sum of values of bidding games with one-type asset.

2) if share prices have finite variances, then the values of n-stage games do not exceed one half of the sum of variances of share prices. This makes reasonable to consider the bidding of unbounded beforehand duration that are modeled by the games with infinite number of steps.

Here we investigate the multistage bidding with risky assets of two types.

We get the solutions for infinite games with probability distributions over two-dimensional integer lattice having finite component variances. The value of such game is equal to the sum of component variances for distributions with integer component expectations and its linearization for other distributions. Therefore, this is equal to the sum of values of games with one risky asset (see [2]). Thus, the profit that Player 2 gets under simultaneous n-step bidding in comparison with separate bidding of each type of shares disappears in the game of unbounded duration.

Both players have optimal strategies. The optimal strategy of Player 2 is a direct combination of his optimal strategies for the games with one-type asset.

For constructing the optimal strategy of Player 1 we get the canonical symmetric decomposition of probability distributions over two-dimensional integer lattice with a fixed integer expectation vector, into a convex combination of extreme points, i.e. distributions with not more than thre-point supports.

The optimal strategy of Player 1 for the bidding game for shares of two types with arbitrary distribution is the convex combination of his optimal strategies for such games with distributions having not more than three-point supports. The last are the elements of the symmetric representation of the initial distribution.

The martingale of posterior mathematical expectations generated by the optimal strategy of Player 1 for the game with the three-point support distribution represents a symmetric random walk over points of integer lattice lying within the triangle spanned over the support points of distribution. The symmetry is broken at the moment when the walk hits the triangle boundary. Starting at this moment, the game get over one of games with distributions having two-point supports.

References

[1] Kreps V., Domansky V. Multistage biddings with several risky assets. International Conference Game Theory and Management GTM2009. Abstracts. Eds. by L,Petrosjan and N.Zenkevich, 2009, 135-136.

[2] V.Domansky, V.Kreps "Multistage biddings with risky assets: the case of countable set of possible liquidation values" In: Game Theory and Management, GTM2007, Collected papers. Ed. by L,A.Petrosjan and N.A.Zenkevich, p.92-106.

Ordinally Equivalent Power Indices

Josep Freixas[1] and **Montserrat Pons**[2]

[1,2]*Escola Politecnica Superior d'Enginyeria de Manresa, Spain*
[1]*josep.freixas@upc.edu*
[2]*montserrat.pons@upc.edu*

Keywords: *Power indices, Values, Voting, Rankings, Orderings*

Different reasonable power indices provide different orderings of importance for voters, so that the evaluation of (a priori) power in (binary) voting systems is quite arbitrary since the rankings highly depend on the particular power index chosen. One is left with the hope that such discrepancies occur because the voting system at hand is rare enough. To better understand what happens, we analyze the ordinal equivalence of families of power indices. These families include the following indices: Shapley-Shubik, Banzhaf, the Johnston, Rae, Coleman to prevent, and Coleman to initiate.

Power indices based on symmetric probabilistic values (or semivalues) with positive coefficients (regular semivalues), as the Banzhaf index or the Shapley-Shubik index, share the same rankings of voters within the class of weakly complete games. Weakly complete games contain weighted and complete games and, therefore, the most common real voting systems. This partially solves the problem because power indices are mostly applied to non-complicated voting systems derived from real problems which always are weakly complete. However, the analytical problem of studying the ordinal equivalence, even for regular semivalues, is still not solved outside weakly complete games. We give an extension which implies the ordinal equivalence of the Banzhaf and Shapley-Shubik indices (and also of the Rae and the two Coleman indices).

The Johnston index also shows a good behavior since it is ordinally equivalent to Banzhaf and Shapley-Shubik indices in a sufficiently large class of games containing complete games. Necessary and sufficient conditions are given to determine such a class. Oppositely, families of power indices based only on minimal winning coalitions, as Holler, Deegan-Packel or the Shift indices, show a non-desirable ranking behavior.

Resale in Auctions with Financial Constraints

Maria Angeles De Frutos[1] and **Maria Paz Espinosa**[2]

[1]*Universidad Carlos III de Madrid*
Spain
frutos@eco.uc3m.es
[2]*Universidad del Pais Vasco*
Spain
mariapaz.espinosa@ehu.es

Keywords: *Auctions, Subcontracting, Financial constraints, Resale, Asymmetric information*

This paper analyzes auctions where bidders face financial constraints that may force them to resell part of the property of the good (or subcontract part of a project) at a resale market. We show that the interaction between resale and financial constraints changes previous results on auctions with financial constraints and those in auctions with resale. The reason is the link between the resale price and the auction price introduced by the presence of financial constraints. Such link induces a potential loser to increase the auction price in order to fine-tune the winner's resale offer, which may require forcing the winner to be financially constrained. We characterize equilibria and show how the auction price is related to the buyers' valuation considering the resale stage.

Competition for acquiring a public firm or winning the allocation of a big facility is often characterized by the presence of a small number of qualified bidders who assign a large value to the good although they may face financial constraints. Because of this, the acquirer can share the property of the good with other buyers. One specific example is the allocation problem of the European Spallation Source that has to be allocated to a single country or location but whose property can be shared after the initial allocation, to alleviate the winner's financial constraints. Similarly, operating licences (e.g., in the telecommunications sector) are awarded to one firm, and (some or all of) the actual services can be subcontracted. Horizontal subcontracting is a common phenomenon in many industries (see, e.g., Kamien et al., 1989; Spiegel, 1993; and Chen et al., 2004, for further discussion and examples). There is horizontal subcontracting or horizontal outsourcing whenever a fraction of the final production of a good is subcontracted to rival firms. Beyond these particular examples, this framework fits privatization processes involving the sale of a public firm to a single buyer meeting the (legal, administrative) requirements or the procurement of large-scale production contracts in the public sector. Previous work on auctions with resale relies mostly on the potential inefficiencies of the auction allocation mechanism to provide the basis for resale. An inefficient allocation may result from noisy signals at the time of the auction, as in Haile (2000, 2001, 2003), from bidders' asymmetries when the auction is conducted as first price, as in Gupta and Lebrun (1999) or Hafalir and Krishna (2008), or from the presence of speculators who value the object only by its resale price, as in Garratt and Tröger (2006). In contrast, in our model the auction is a second price auction and there are no pure speculators, all participants value the good, and the resale market is justified by the presence of financial constraints which may force the winner of the auction to sell part of the property of the good. That is, if bidders were wealthy enough the resale market would be inactive and it is the presence of financial constraints which makes participants aware that there will be partial resale.

The literature on auctions with resale has focused on the case of total resale of the good and to the best of our knowledge this is the first paper introducing partial resale in an auction framework under incomplete information. However, partial resale (or horizontal subcontracting) is a common assumption in two-stage contract games under other modes of competition. Kamien, Li and Samet (1989) study a procurement auction for an endogenously determined quantity of a perfectly divisible good with two identical and completely informed bidders. They show that decreasing returns create a need for subcontracting, the same role played by financial constraints in our model. Our model contributes to the literature on horizontal subcontracting where the object to be procured has a fixed size, while relaxing the important assumptions of symmetry and complete information. By considering a double source of asymmetry (in use-values and in wealths) our paper is also close to that of Spiegel (1993) where firms are supposed to compete in quantities rather than prices again under complete information. As in our model, incentives to resale arise from asymmetries and at the bidding stage firms take advantage of their (relative) strength. Finally, another related paper is Meland and Straume (2007) who analyze outsourcing in contexts in which the final allocation may depend on the effort undertaken by competing suppliers; as it is also the case in our model, they find that resale (outsourcing) increase aggregate firms profits due to improved allocative efficiency. Nevertheless two important differences are noteworthy: first, we consider a second price auction and not an all-pay auction; second, we assume that use-values are private information whereas they analyze a complete information context.

The main purpose of the paper is to highlight the link between the resale price and the auction price introduced by the presence of financial constraints. Such link induces a potential loser to increase the auction price in order to fine-tune the winner's resale offer, which may require forcing the winner to be financially constrained. We first compare the outcomes of the auction with resale and financial constraints, to those without resale or without financial constraints, and find that the loser's incentives to modify the resale price may preclude a truth telling behavior and also bidding the minimum between valuation and wealth as part of the equilibrium. We also find that the presence of financial constraints may eliminate the speculative equilibria à la Garrat and Tröger in auctions with resale. The paper is organized as follows. Section 1 presents the model, which is solved in Section 2 under complete information. In Section 3 we solve the resale stage and the bidding stage under private information on use values and present the main results of the paper. Section 4 concludes.

Meeting up on a Network

Edoardo Gallo

University of Oxford.
United Kingdom
edoardo.gallo@economics.ox.ac.uk

Keywords: *Network, Speed of convergence, Learning, Mean preserving spread*

The structure of the environment individuals navigate determines the opportunities they have to interact, a prerequisite for many economic processes. A network can provide a very general representation of the structure of an environment: the nodes are the possible states individuals can find themselves in and individuals' probabilities to transition to different states are determined by the links in the network. Here we formulate a general framework to investigate how an environment, modeled by a network, determines the mean time T an individual has to wait between two meetings. We derive an expression to compute, for any given network, how T is determined by the underlying network structure, and we show how T changes if we vary the network by a mean-preserving spread of the degree distribution. If the number of individuals is low (high), then a mean-preserving spread of the degree distribution of the network reduces (increases) the expected time between meetings. The applicability of this framework is illustrated by two stylized models of trading and learning. In the trading model T determines traders' outside option in bilateral bargaining. An extension shows how T changes if we introduce capacity constraints to the number of traders that can occupy a node in the network. In the learning model T determines the speed of convergence to the truth about an unknown state of the world. An extension shows how the presence of a small number of influencers, individuals with an extreme view and a tendency to gravitate toward more connected nodes, pulls a society away from the truth.

A Very Stable Cooperative Solution for n-Person Games

Gianfranco Gambarelli

Department of Mathematics, Statistics,
Computer Science and Applications,
University of Bergamo, Italy
gambarex@unibg.it.
http://dinamico.unibg.it/dmsia/staff/gambar.html

Keywords: *Characteristic Function Form, Normal Form, Pareto-optimality, Nash-equilibria*

The Problem

Various management cases may be described by mean of games in characteristic function form: see for instance Gambarelli (1982), and Gambarelli and Owen (1994). It is interesting for the manager to be aware not only of the resulting solutions, but also of the behaviour patterns that may lead him to them. However, this strategic aspect is lacking in the classical solutions for games in characteristic function form (core, values and so on). In fact, the latter are "coalition-oriented", i.e., they are constructed solely from the viewpoint that each coalition must satisfy all its members.

The Approach

The approach proposed by Gambarelli (2007) enables strategies to be placed in the hands of the decision-makers and therefore gives a "man –oriented" solution, which is generally different from classical ones. The model is developed on the basis of a transformation of the original game, in characteristic function form, into a corresponding game in normal form, thereby generalizing an analogous transformation introduced by von Neumann and Morgenstern for finite superadditive constant-sum games (see pp. 238-245 of the third edition).

The Transformation

The transform is carried out as follows. Each player, in a preliminary examination of the game, asks himself the question: how much should I demand as my price for taking part in each coalition? If I ask too little I will be unable to optimise my winnings, while if I ask too much the coalition will reject me. More precisely, the more I demand, the less probability I have of belonging to a coalition which will actually be formed, leading to the extreme case in which I belong to no such coalition and therefore can obtain nothing. The strategy profiles of each player are his claims, while the (expected) payments consist of these claims multiplied by the probability that the player belongs to a coalition that will actually be formed.

The Solution

The solution for the transformed game is defined as the set of Nash equilibria leading to Pareto optimal payments. Such payments are considered to be the solution of the original game. In this sense the solution can be considered very stable.

Existence and Uniqueness

A solution constructed in this way is not generally unique. Existence has been proved for a wide range of games: all the inessential games, all the subadditive games, all the 2-person games and, regarding equiprobable coalition games, all the 3-person simple games and all the n-person games with a non-empty interior of the core. A game lacking such a solution has yet to be found.

In this talk

Various questions thus remain open, including the search for solutions for the remaining classes of games (or counter-examples of non-existence) and of conditions for the uniqueness of the corresponding imputations. In this talk, some ideas will be presented as a response to the foregoing observations. Put briefly, conditions of constraint or relaxation of the solutions of the transformed game are being studied, in order to guarantee uniqueness and possibly existence. The solution thereby obtained might well become a very stable game value" of the original game in characteristic function form.

References

[1] Gambarelli, G. (1982) "Portfolio Selection and Firms' Control" Finance, pp. 3, 1, 69-83.

[2] Gambarelli, G. (2007) "Transforming Games from Characteristic into Normal Form" International Game Theory Review, pp. 9, 1, 87-104.

[3] Gambarelli, G. e G. Owen (1994) "Indirect Control of Corporations", International Journal of Game Theory, pp. 23, 4, 287-302.

[4] von Neumann, J., and O. Morgenstern (1944) Theory of Games and Economic Behavior, 3rd ed. 1953. Princeton, NJ: Princeton University Press.

Numerical Study of a Linear Differential Game with Two Pursuers and One Evader

Sergey Ganebny[1], **Sergey Kumkov**[2], **Valery Patsko**[3] and **Stephane Le Menec**[4]

[1,2,3]*Institute of Mathematics and Mechanics,*
Ural Branch, Russian Academy of Sciences
Russia,
[1]*nordwinder@gmail.com*
[2]*2445@mail.ur.ru*
[3]*patsko@imm.uran.ru*
[4]*EADS / MBDA France*
slemenec@free.fr

Keywords: *Linear differential games, Two pursuers and one evader game, Maximal stable bridges*

Abstract: *A linear game on a plane with two pursuers and one evader is considered. The work deals with a numerical study of the problem based on maximal stable bridge construction.*

In work [1], a problem with two pursuers $P1$ and $P2$ and one evader E is considered. It is supposed that all three objects move in some fixed plane. Evader heads the pursuers. Initially, velocities of all objects are parallel. The case is most interesting when the evader is located between the pursuers. The closing velocities are significantly great, but the control lateral accelerations are quite small, so the passing instants can be considered as fixed. Assuming the initial instant as zero, denote the passing instants by T_{f1} and T_{f2}.

A model problem of this type can arise when studying a pursuit of an aircraft by two others in the horizontal plane.

In difference coordinates, the linearized dynamics looks like

$$\begin{aligned} \ddot{y}_1 &= -a_{P1} + a_E, & \ddot{y}_2 &= -a_{P2} + a_E, \\ \dot{a}_{P1} &= (A_{P1}u_1 - a_{P1})/l_{P1}, & \dot{a}_{P2} &= (A_{P2}u_2 - a_{P2})/l_{P2}, \\ \dot{a}_E &= (A_E v - a_E)/l_E. \end{aligned}$$

Here, y_1 and y_2 are the lateral deviations of the first and second pursuers from the evader; a_{P1}, a_{P2}, a_E are the current lateral accelerations of the pursuers and evader;

u_1, u_2, v are the controls; A_{P1}, A_{P2}, A_E are the maximal value of accelerations; l_{P1}, l_{P2}, l_E are the time lag constants. All controls have bounded absolute values:

$$| u_1 |\leq 1, \quad | u_2 |\leq 1, \quad | v |\leq 1.$$

In [1], a case is considered when each pursuer is stringer than the evader. This means that if at some instant the lateral deviation of the evader form a pursuer becomes zero then this pursuer provides exact capture. The game objective is to capture exactly the evader by at least one pursuer.

For the case $T_{f1} = T_{f2}$, an analytic description is suggested in [1] for the set of all initial positions, from which the game can be successfully solved. With that, a passage to two equivalent coordinates x_1 and x_2 is applied. Each of these coordinates is one-dimensioned and describes the forecasted lateral deviation at the passing instant. Also, some reasonings are given for the case $T_{f1} \neq T_{f2}$, which is more difficult. The most difficult situations (from the point of view of analytical investigation) are when one or both pursuers are weaker than the evader.

This work deals with a numerical study of the problem. Numerical procedures for constructing maximal stable bridges [2] are involved. If $T_{f1} = T_{f2}$, we meet the problem of constructing a stable bridge in a linear game with fixed terminal time and non-convex cross-like terminal set. If $T_{f1} \neq T_{f2}$, then the problem is of guiding the system to a target set, which consists of the axis Ox_1 at the instant T_{f2} and the axis Ox_2 at the instant T_{f1}.

As a result of the investigation, the case is completely studied when both pursuers are stronger than the evader and $T_{f1} \neq T_{f2}$. Pictures of time sections of the stable bridges are given.

References

[1] Le Ménec S. Linear Differential Game with Two Pursuers and One Evader. In: Abstracts of 13th International Symposium on Dynamic Games and Applications. Wroclaw, Poland, 2008. P. 149 - 151.

[2] Krasovskii N.N., Subbotin A.I. Game-Theoretical Control Problems. Springer, New York, 1988.

The Dynamic Procedure of Information Flow Network*

Hongwei GAO[1], **Yeming DAI**[2], **Wenwen LI**[3], **Lin SONG**[4] and **Tingting LV**[5]

Qingdao University, College of Mathematics,
Ningxia Road, 308, 266071, Qingdao, China
[1]*cmgta2007@163.com,*
[2]*yemingdai@163.com,*
[3]*a-liwenwen@163.com,*
[4]*slqsyq@163.com,*
[5]*joelys0505@126.com*

Keywords: *Network formation games, Core-periphery architecture, Dynamic procedure*

We observe the process and algorithm design of the dynamic change from information flow network to core-periphery network and program a dynamic procedure by Maple. The problem is one of the two main concerns that Goyal S. proposed in his recent research[3]. In this paper any network is chosen as the initial network, the player adopts optimum reaction in order specifically for current network, equilibrium network is formed finally only when the network structure and information volume of the player have reached a steady state. Based on the established model, a new incomplete cooperative information flow network game model which algorithm and procedure of dynamic generation are achieved will appear.

While inspecting the convergence process of dynamic procedure, we also give a non-equilibrium example with stable topology structure, where the information content of players with positive information moves endlessly in circles. The conditions of the particular case need to be followed closely.

At the same time a network game model about global greenhouse gas emissions based on bilateral quota trade is proposed. Because in fact they need more greenhouse

*This work was partially supported by the National Natural Science Foundation of China (Grant No.70571040,70871064), the International (Regional) Joint Research Program of China (Grant No.70711120204)
and the Innovation Project of Graduate Education in Shandong Province (Grant No.SDYC08045)
Corresponding author Hong-wei GAO E-mail: gaosai@public.qd.sd.cn

gas emission quota than what reality gains, developed country plan to set up bilateral trade with other countries holding quota when they shoot for quota. We try to set up network game model based on the multilateral quota trade; of course it is also helpful to carry out the research simulating the trade.

MSC(2000): 91D30 91A25

References

[1] Jackson M O, Wolinsky A. A strategic model of social and economic networks□Journal of Economic Theory, 1996, 71(1), pp.44-74

[2] Bala V, Goyal S. A non-cooperative model of network formation. Econometrica, 2000, 68, pp.1181–1229.

[3] Goyal S□Galeotti A. The Law of the Few. American Economic Review, 2009, forthcoming

[4] Bala V, Goyal S. A strategic analysis of network reliability. Review of Economic Design, 2000, 5, pp.205–228.

[5] Derks J, Kuipers J, Tennekes M, Thuijsman F. Local Dynamics in Nash network formation. Maastricht University, Department of Mathematics, 2008.

[6] Derks J., Tennekes M. A note on the existence of Nash networks in one-way flow models. Economic Theory, 2009, 41(3), pp.515-522.

[7] Derks J, Kuipers J, Tennekes M, Thuijsman F. Existence of Nash networks in the one-way flow of network formation. In: Modeling, Computation and Optimization, Eds S.K. Neogy, A.K. Das, R.B. Bapat, World Scientific, 2009, pp.9-20.

[8] Galeotti A. One-way flow networks: the role of heterogeneity. Economic Theory, 2006, 29(1), pp.163–179.

[9] Haller H, Sarangi S. Nash networks with heterogeneous links. Mathematical Social Sciences, 2005, 50(2), pp.181–201.

[10] Myerson R B. Graphs and cooperation in games. Mathematics of Operations Research, 1977, 2, pp.225-229.

[11] Gao H., Petrosyan L.A. Dynamics Cooperative Games. Science Press, Beijing, 2009 (in Chinese).

[12] Schweitzer F, Fagiolo G, Sornette D, Vega-Redondo F, Vespignani A, White D R. Economic Networks: The New Challenges. Science, 2009, 325, pp.422-425.

Models of Network Formation Game Control

Mikhail V. Goubko

Institute of Control Sciences of Russian Academy of Sciences,
Moscow,Russia
mgoubko@mail.ru

Keywords: *Organizational control, Network formation game, Discrete optimization*

Abstract: *In a basic model of organizational systems control theory a system is considered where a sole principal controls several agents. In a wide range of practical situations the agents try to build some network structure by establishing bonds to each other. The principal's role (the control problem) is to force formation of certain bonds between agents. Given the control action the agents play a network formation game (NFG). Control theory imposes strong requirements on the game-theoretic solution concepts employed. Traditional solution concepts, like Nash equilibrium, when applied to NFGs, fail to satisfy these requirements. Thus, several special strategic solution concepts were developed in the literature for NFGs. In the report these concepts are compared and aligned into the line from the weakest to the strongest A general setting is considered of incentive problem where the principal can immediately set the bonus to the agents for the formation of specific networks. This problem is solved in complete information framework for several NGG solution concepts.*

In a basic model of organizational systems control theory [8] a system is considered where a sole principal controls $n \geq 1$ agents. The principal chooses a control $u \in U$ to maximize his/her utility function $\Phi(x,\ u)$, where $x \in X$ is a state of the system. Each agent has a utility function $f_i(y_1,...,\ y_n,\ u)$ that can depend on his or her action $y_i \in Y_i$, actions of others agents, and the principal's control $u \in U$. Agents' actions vector $y_1,...,\ y_n$ along with the control u (and, in general, some uncertain external parameters $\theta \in \Omega$) leads to the result $x(y_1,...,\ y_n,\ u,\theta)$. Thus, given the control u agents play a game with the outcome x. If the principal uses some game-theoretic solution concept $P(u) \subseteq X$ to predict the outcome of the game, then the control problem is to find the optimal control $u \in Arg\max_{u \in U} \min_{x \in P(u)} \Phi(x,u)$. The set U may allow for loopback controls $u(x)$ if they make sense in a considered setting.

If agents are involved in some network interaction (choose a path through a transportation network [5], participate in a process of intercommunication in a social network [10], or share information with colleagues in models of information security [2], etc), the output x describes the resulting parameters of this network (equilibrium traffic densities in networking games, sustained opinion in the models of intercommunication …). Specific solution concepts (like Wardrop equilibrium or information equilibrium [5]) allow predicting the rational actions of agents. In these models the principal influences agents' behavior by directing traffic in transportation network, injecting the judgments into a social network, or choosing the information security policy.

In a wide range of problems the agents form the network structure itself. For example, in the problem of road building investment agents must agree on the structure of a roads system and then share the construction costs among the parties. Agents choose the circle of acquaintance (and then communicate with the neighbors) in the models of social networks, or support the communication channels in the models of local network security. The principal role is to force formation of certain bonds between agents by tax policy or public-private partnership agreements in the problem of road investment, by promoting some types of bonds in social network, by limiting access in the problem of information security, etc.

Often the framework of network formation employs some model of network interaction (game-theoretic or simulation, and sometimes rather complicated) to calculate the agents' payoffs, but to stress the structure formation aspects it is supposed below that agents' payoffs $f_i(y_1,..., y_n, u)$ depend solely on their network formation efforts yi and on control variable u. The state of the system x is a network – a directed or undirected graph built over the set of agents (some extensions involve the vector weights attached to the edges of the graph), and the problem of the principal is to find a control maximizing his or her utility function $\Phi(x, u)$.

The tradition treats network formation game (NFG) as a generalization of cooperative game in characteristic function form, although NFGs admit purely strategic analysis. The mechanism of network formation (i.e. the relation between the agents' efforts $y_1,..., y_n$ and the resulting network $\mathrm{x}(\cdot)$) can be very complicated. Typical mechanisms involve mutual agreement on bond formation (for example, both agents must agree to become friends), and unilateral formation of directed arc (usually no approval is required from recipients of local corporate e-mails). In the former the actions set of every agent includes the binary vector of offers and the vector of acceptance of other agents' offers, while in the latter a binary vector of intentions is enough.

Control theory imposes strong requirements on the solution concepts employed [8]. Existence of solution guarantees the predictability of agents' behavior, while the uniqueness (or narrowness) of solution concept assures the controllability of the system. Traditional solution concepts, like Nash equilibrium, when applied to NFGs, fail to satisfy these requirements (for example, the empty network is a Nash equilibrium of mutual agreement game irrespective of agents' payoffs). Thus, several special strategic solution concepts were developed in the literature for NFGs (see the survey in [11]): pairwise stability [12], strong Nash stability [3], hybrid stability, and k-stability [6].*

All these solution concepts have their own pluses and minuses. In the report I discuss and compare these concepts, align them into the line from the weakest to the strongest. This allows the principal to choose the concept that describes best agents capabilities in a specific setting.

Rather common control problem is the problem of maximizing the total value of network – the sum of agents' payoffs. Although in this case the goals of principal do not conflict with the goals of controlled system, the problem of stabilizing the effective network is far from trivial, and the principal has to reallocate the payoff among the agents in a very sophisticated way to resolve the conflict of efficiency and stability [4,11].

In the report a more general setting is considered of incentive problem where the principal can immediately set the bonus to the agents for the formation of specific networks [7]. This problem is solved in complete information framework for several solution concepts including pairwise stability, strong stability, and hybrid stability. The problem is reduced to the set of discrete optimization problems, their complexity is estimated, and in some cases closed form solutions are obtained. Some applications of the theoretic results are discussed.

References

[1] Bala V. and Goyal S. A Strategic Analysis of Network Reliability // Review of Economic Design. 2000. N 5. P 205–228.

[2] Danilenko A.I. Information control over an organizational system with the possibility of information leakage // Collected papers of 51th scientific conference of MIPT. Moscow: MIPT. - 2008.

[3] Dutta, B., Mutuswami S. Stable Networks // J. Econom. Theory. 1997. N 76. P 322–344.

* Another branch of literature considers cooperative solutions of network formation games studying the so called, allocation rules for NFGs *[4] (*the generalization of imputation notion*)*.

[4] Dutta B., van den Nouweland A., Tijs S. Link Formation in Cooperative Situations // Int. J. Game Theory. 1998. N 27. P 245–256.
[5] Florian M., Hearn D. Network equilibrium models and algorithms / Network Routing. NY: Elsevier Science, 1995. P. 485-550.
[6] Goubko M. Control of Organizational Systems with Network Interaction of Agents. I. A Review of Network Game Theory // Automation and Remote Control. V. 65, No 8. 2004. 1276-1291.
[7] Goubko M. Control of Organizational Systems with Network Interaction of Agents. II. Stimulation Problems // Automation and Remote Control. V. 65, No 9. 2004. 1470-1485.
[8] Goubko M., Novikov D. Game-theoretical Models and Methods of the Organizational Systems Control Theory // Game Theory and Management. Collected abstracts of papers, presented in the International Conference "Game Theory and Management", Gradual School of Managemant, SPbU, 2008.
[9] Goubko M.V., Novikov D.A., Chkhartishvili A.G. Network Games and Games over Networks // Networking Games and Management. Extended Abstracts. Petrozavodsk: IAMR KRC RAS. 2009. 13-17.
[10] Gubanov D.A., Novikov D.A., Chkhartishvili A.G. Socialniye seti: modeli informacionnogo vliyaniya, upravleniya I protivoborstva. Moscow, Fizmatlit. 2010.
[11] Jackson M. Social and Economic Networks. – Princeton: Princeton University Press, 2008.
[12] Jackson M., Wolinsky A. A Strategic Model of Social and Economic Networks // J. Econom. Theory. 1996. N 71. P 44

ВЕСТНИК САНКТ–ПЕТЕРБУРГСКОГО УНИВЕРСИТЕТА

Серия 8
МЕНЕДЖМЕНТ

Ответственный редактор серии:
Ю.Е. Благов

Издательство Высшей школы менеджмента Санкт-Петербургского университета

A Game Theoretical Study of the Wheat Market in South Italy

Luca Grilli[1] and **Angelo Sfrecola**[2]

[1]*University of Foggia,*
Italy
l.grilli@unifg.it

[2]*Dipartimento DSEMS*
Italy
a.sfrecola@unifg.it

Keywords: *Wheat market, Leader-Follower, Public incentives*

In this paper we investigate, in a game theoretical context, the wheat market in South Italy. In particular we consider the ‘‘Tavoliere’’ area in the Region Puglia, this area is also known as ‘‘granaio d'Italia*‘‘. The ‘‘Tavoliere’’ economy is based on agriculture and in particular wheat-growing. Typically the farmer is also the owner of the land. The farmer has to choice the wheat variety to raise and consequently the quantity to be produced according to land availability. The selection of wheat variety is crucial since the European Community provides incentives only if the wheat produced satisfies specific characteristics in terms of protein, gluten and others. Once the wheat has been harvested the farm could sell it in the market at the current price which is determined in a specific Exchange. Nevertheless, the farmer uses to sell the wheat produced to an intermediate trader that will sell it, later, in the market. This is due to impossibility, for the farmer, to stock the harvested wheat and wait for selling it since small farmers, in this area, do not have grain bins. In this paper we consider a model in which there are two agents: the trader and the farmer. We suppose that they interact following a Leader-Follower structure, where the trader is the Leader and the farmer is the Follower. The farmer has to decide the quantity to sell considering also the Communitarian incentives, as a best reply to the announced price asked by the trader.

Subject Classification: 91A35; 91B32; 91A65.
JEL Classification: C72; Q10; Q31.

* Italian Granary

Cash Flow Optimization in ATMs Network Model

Elena Gubar[1], **Maria Zubareva**[2] and **Julia Merzljakova**[3]

Saint-Petersburg University
Russian Federation
[1]*alyona_kor@yahoo.com*
[2]*becks88@yandex.ru*
[3]*juulkin@rambler.ru*

Keywords: *ATMs network, Optimal routes, Vehicle Routing Problem, Cash flow, Statistical methods*

Abstract: *The main purpose of this work is to optimize cash flow in case of the encashment process in the ATMs network with prediction of ATMs refusal. The solution of the problems is based on some modified algorithms for the Vehicle Routing Problem and use statistical methods to compile the requests from the ATMs network. A numerical example is considered.*

In this work we consider a problem in which a set of geographically dispersed ATMs with known requirements must be served with a fleet of money collector teams stationed in the depot in such a way as to minimize some distribution objective. This problem is combined with the problem of composition service requests from the ATM network. We assume that the money collectors teams are identical with the equal capacity and must start and finish their routes at the depot. To estimate an average cash amount in each ATM and form the requests for the money collectors team we use following factors:

- ATM locations and working time;
- Irregular number of operations per week;
- Increasing of operations number in holidays and salary days.

Moreover we define the necessity of servicing each ATM and predict the future requests for the collectors teams, based on the statistical data and restrictions, which are proposed above. The optimal routes to load ATMs depend on the current requests and predictable requests.

The main purpose of this work is to optimize cash flow in case of the encashment process in the ATMs network with prediction of ATMs refusal. To solve the problems we base on some modified algorithms for the Vehicle Routing Problem and use statistical methods to compile the requests from the ATMs network. A numerical example is considered.

References

[1] Hamdy A. Taha. Operations Research: an Introduction. University of Arkansas, Fayetteville: Pearson Education, Inc.,2003.
[2] Randolph W. Hall. Handbook of Transportation Science. Springer,2003.
[3] Christofides N., Mingozzi A., Toth P. Exact Algorithms for theVehicle Routing Problem, Based on Spanning Tree and Shortest Path Relaxations. Mathematical Programming, 20, 255-282,1981.
[4] Lysgaard J., Letchford A., Eglese R. A New Branch-and-Cut Algorithm for the Capacitated Vehicle Routing Problem. Aarhus School of Business, Denmark, 2003.
[5] Bure V.M., Evseev E.A. Basis of econometrics. Saint Petersburg State University, 2004.
[6] Vasin N.S. The Mathematical-statistical Model for Accounting of Payment Transactions in ATM Networks. Discount and TaxPolicy: Planning and Development Prospects: Materials of All-Russian Theoretical and Practical Conference. Orel, 2005, pp. 215-217

On Managing Depositary Bank Risk-based Reserve

Anton Gurevich

Faculty of Applied Mathematics and Control Processes
St. Petersburg University
Universitetskii prospekt 35, Petergof
St. Petersburg, 198504, Russia
anton.gurevich@gmail.com

Keywords: *Risk management, Banking firm, reserve, Bankruptcy, Operational measure of riskiness, Deposit bank.*

In this report the problem of achieving stable activities of the depositary bank by controlling the volume of its reserves is considered. For this we construct a model of the depositary bank and show how to change the distribution of the random variable characterizing its profits with the change of reserve. Next, using an operational measure of risk, described in [1], we consider how to change the riskiness of the random variable obtained depending on the reserve. Finally, we show the risk management strategy based on bank reserves, which, if it is theoretically possible, brings avoiding bankruptcy for the bank.

References

[1] Foster D.P., Hart S. (2007) "An operational measure of riskiness" http://www.ma.huji.ac.il/hart/papers/risk.pdf.

A Piecewise Deterministic Game Model for International GHG Emission Agreements*

Alain Haurie

University of Geneva,
Place de l'Etrier 4
CH-1224 Chêne-Bougeries
Switzerland
Tel: +41 22 348 2046
Fax: +41 22 348 2083
http://www.ordecsys.com
and
HEC Montreal,
Montreal, Canada

This presentation concerns a class of stochastic games that can be used to assess the impact of uncertainty in the negotiation of self enforcing international environmental agreements (IEA) among different groups of countries. The uncertainty in the model concerns the access, through R&D investment, to a new backstop technology permitting a clean generation of energy and also the possible occurrence of catastrophic climate regime change triggered by the accumulation of GHGs. The negotiation is represented by a cost-benefit analysis performed on optimal economic growth models for two groups of countries. The uncertainty is modeled as a controlled jump process. The IEA is represented by a Nash equilibrium in the class piecewise open-loop strategies for a piecewise deterministic differential game. The presentation is focused on the numerical method implemented to solve the large-scale dynamic programming equations characterizing Nash equilibrium solutions. The results are interpreted in terms of "optimal timing" of abatement policies in developed or developing countries.

* Joint work with O. Bahn, Associate Professor, GERAD and MQG, HEC Montreal, Montreal, Canada, and J. Theni research associate at ORDECSYS, Geneva. The author acknowledges financial support by: EU-FP7 PLANETS and GICC (French Ministry of Ecology and Sustainable Development).

Signaling Managerial Objectives to Elicit Volunteer Effort

Gert Huybrechts[1], **Jurgen Willems**[1], **Marc Jegers**[1], **Jemima Bidee**[2], **Tim Vantilborgh**[2] and **Roland Pepermans**[2]

[1]*Micro-economics for Profit and Non Profit Sector*
[2] *Work & Organizational Psychology*
Pleinlaan 2 - 1050 Brussels - Belgium
Tel: +32.2.629.22.14
gert.huybrechts@vub.ac.be

Keywords: S*ignaling, Volunteers, Nonprofit, Motivation*

Volunteers are an important resource for nonprofit organizations (npos). Therefore, appropriate management of volunteers can help nonprofit organizations to achieve their mission. However, volunteers will not blindly obey any instruction given by the management, as they are also rational utility optimizing agents. If volunteers are to exert effort then they must consider this as their optimal choice. Formal control systems may be problematic because they undermine the employees' intrinsic motivation (Frey, 1997; Frey and Jegen, 2001). Our paper may offer an explanation for 'crowding-out' of intrinsic motivation. If the absence of formal control systems in a npo is viewed by the volunteer as a positive signal of the manager's objectives, the volunteer may prefer higher effort when there is no formal control system. In this perspective our paper is similar to Bénabou and Tirole (2003) who investigate the effects of rewards on the agent's motivation. Rewards may have a hidden cost if they serve as a signal that reduces effort. Not only rewards serve as signals. Every action by the manager can be a signal if it satisfies two requirements: 1) the action is observed by the volunteer 2) the manager's whose objectives are most appreciated by the volunteer has the lowest signaling costs.

We construct a signaling model (Spence, 1973) of a npo involving a manager (she) and a 'motivated' volunteer (he). We assume there are two types of managers, the identical type who shares the same goals as the volunteer and the non-identical type who does not share the same goals. The order of the game is as follows: Nature chooses the type of the manager which is not revealed to the volunteer. The manager can send a signal to the volunteer to convince him that she is of an identical type. Given the signal, the volunteer chooses his optimal amount of effort.

The volunteer is by definition unpaid, but he can receive intrinsic rewards from the output that is produced by his effort (Besley and Ghatak, 2003), thus satisfying a number of functions that serve as the volunteer's goals e.g. values, understanding, career, enhancement, social and protective functions (Clary et al., 1996). The volunteer's goals are the functions that he wants to satisfy. Whether or not the volunteer is able to satisfy these goals when working for the npo, depends on the type of manager. The volunteer only prefers high effort when he is working for an identical manager. The manager can try to signal that her goals are the same as the volunteer's.

The signal is assumed to consist of two parts: the internal and the external signal. The internal signal is under control of the manager and can be increased at a cost. Human resource management policies (Guzzo and Noonan, 1994), mission statements or advertisements (Rousseau, 1995) can be viewed as part of the internal signal. The external signal is not under control of the manager but is sent by external actors such as other agents of the npo (recruiters, co-workers or former volunteers) or other stakeholders (donors, media, etc.). The external signal can even be unintended signals sent by the manager (Rousseau, 1995). We assume the external signal to be different for each type of manager and not necessary in line with the internal signal. This gives the identical manager a cost advantage when sending the signal because she benefits more from the external signal than the non-identical manager does.

In solving this model we find that only three equilibriums remain: 1) a lower bound pooling, 2) a lower bound separating equilibrium, and 3) an upper bound separating equilibrium. Which of these equilibriums is chosen depends on the incentive compatibility constraints of the managers. The lower bound separating equilibrium Pareto dominates the other equilibriums. It is optimal for the identical manager to find a separating equilibrium at the lowest possible signaling cost. To realize this she must determine the optimal signal and the optimal size of the signal. Any action taken by the identical manager can serve as a signal of the manager's type when it meets the following requirements: the action is observed by the volunteer and the identical manager has a relative cost advantage compared to the non-identical manager. This means that if the identical manager wants to move to a more favorable equilibrium, the range of possible actions can be large.

The absence of formal control systems may be a positive signal of managerial objectives if the volunteer can observe it and if the increase in shirking caused by this absence is more costly for the identical manager than for the non-identical manager.

References

[1] Besley, T. & Ghatak, M. (2005), 'Competition and Incentives with Motivated Agents.', American Economic Review 95(3), 616 - 636.

[2] Bénabou, R. & Tirole, J. (2003), 'Intrinsic and Extrinsic Motivation.', The Review of Economic Studies 70(3), 489-520.

[3] Clary, E. G.; Snyder, M. & Stukas, A. A. (1996), 'Volunteers' Motivations: Findings from a National Survey', Nonprofit and Voluntary Sector Quarterly 25(4), 485 - 505.

[4] Frey, B. S. (1997), Not Just for the Money: An Economic Theory of Personal Motivation., Edward Elgar Publishing.

[5] Frey, B. S. & Jegen, R. (2001), 'Motivation Crowding Theory', Journal of Economic Surveys 15(5), 589 - 611.

[6] Guzzo, R. A. & Noonan, K. A. (1994), 'Human resource practices as communications and the psychological contract', Human Resource Management 33(3), 447 - 462.

[7] Rousseau, D. M. (1995), Psychological contracts in organizations: Understanding written and unwritten agreements, Thousand Oaks, CA: Sage.

[8] Spence, M. (1973), Job market signaling.', Quarterly Journal of Economics 87(3), 355 - 374.

МАТЕМАТИЧЕСКАЯ ТЕОРИЯ ИГР И ЕЕ ПРИЛОЖЕНИЯ

Выходит ежеквартально с января 2009 года

Algorithmic Bounded Rationality, Optimality and Noise

Christos Ioannou[1] and **Ioannis Nompelis**[2]

[1]*University of Cyprus,*
Cyprus
christos.a.ioannou@gmail.com
[2]*University of Minnesota,*
United States
nompelis@aem.umn.edu

Keywords*:Moore Machines, Prisoner's Dilemma, Algorithms, Bounded Rationality*

A model of learning, adaptation and innovation is used to simulate the evolution of finite automata (executing strategies), in the repeated Prisoner's Dilemma stage-game. A finite automaton is a mathematical model with discrete inputs and outputs. The specific type of finite automaton used here is a Moore machine (Moore 1956). Moore machines are considered a reasonable tool to formalize an agent's behavior in a supergame (Rubenstein 1986). A Moore machine consists of a finite set of states and a set of transitions from state to state that occur on input symbols chosen from an alphabet.

The simulation is based on a thought experiment. Each of thirty agents is to choose a machine, at random, in order to play the Prisoner's Dilemma stage-game against each other in a round-robin structure. The period-to-period encounters occur with a continuation probability of delta=0.995, for an expected 200 periods. With the completion of all round-matches, the actual scores and machines of every agent become common knowledge. Based on this information, the agents update their machines for the next generation. The update is carried via a genetic algorithm.

Genetic algorithms are evolutionary search algorithms that manipulate important schemata based on the mechanics of natural selection and natural genetics. Thus, genetic algorithms have the capacity to model strategic choices using notions of adaptation, learning and innovation explicitly. The mechanics of genetic algorithms involve copying machines and altering states through the operators of selection and mutation. Initially, reproduction is a process where successful machines proliferate while unsuccessful machines die off. Copying machines according to their payoff or fitness values is an artificial version of Darwinian selection of the fittest among structures. After reproduction, selection results to higher proportions of similar successful machines. On the other hand, mutation is an insurance policy against premature loss of important notions. Even though reproduction and selection effectively search and recombine extant notions, occasionally they may become overzealous and loose some potentially useful material. In artificial systems, mutation protects against such an irrecoverable loss. Consequently, these operators bias the system towards certain building blocks that are consistently associated with above-average performance.

In contrast to previous simulations that assumed perfect informational and implementation accuracy, the agents' machines are prone to errors. Bounded rationality is thus introduced, as random noise. This random noise takes the form of action-implementation errors and perception errors. Implementation errors are errors in the implementation of actions along the lines of Selten's trembling hand; I want to cooperate but (maybe due to pressure), my hand trembles so I choose defection instead. On the other hand, perception errors are errors in the transmission of information; an opponent's cooperation is misperceived as a defection.

The impact of bounded rationality on the agents' machines is then examined under different error-levels. In the first set of computational experiments, the machines are subjected to a constant independent chance of implementation and perception errors of 4%, 2%, 1%, 0.5% and 0%, respectively. In the second set, the errors are affected by an on-off switch. When the switch is on, the machines are subjected to a 4% independent chance of implementation and perception errors, whereas when the switch is off the machines have perfect informational accuracy.

The computations indicate that the incorporation of bounded rationality is sufficient to alter the distribution of outcomes as well as the distribution of the agents' machines. In particular, the evolution of cooperative machines becomes less likely as the likelihood of errors increases. These results are important because they show that the evolution to cooperation is considerably weaker than expected; and the change in the model is ecologically plausible: errors are common in strategic situations. In addition, the prevailing (surviving) machines tend to be less complex as the likelihood of errors increases, where complexity is measured by the average number of accessible states. If one considers the measures used in the existing literature to characterize basic statistical properties of the equilibria, then the results are striking.

Furthermore, the computational experiments indicate that machines in the noisy environments tend to converge faster to a prevailing structure than in the error-free environment. Nevertheless, the prevailing structures in all conditions tend to exhibit low cooperation-reciprocity and low tolerance to defections. In addition, the prevailing structures contain more defecting than cooperating states, with the understanding that the cooperating states are meant to establish some consistent mutual cooperation, whereas the defecting states are meant as punishment-counters in case of non-conformity to the cooperative outcome by the opponent. Finally, in the presence of (as low as) 4% likelihood of errors, open-loop machines (independent of the history of play) emerge endogenously. The latter, cooperate for one period and defect for the next four periods without paying any consideration to the game-plan of the opponent.

Solution of the Hotelling Poblem of Spatial Competition in Secure Strategies

Mikhail Iskakov[1] and **Alexey Iskakov**[2]

[1]*Trapeznikov Institute of Control Sciences RAS,*
Moscow, Russia
mih_iskakov@mail.ru
[2]*UCLA, Los Angeles,*
California, USA
isk_alex@mail.ru

Keywords: *Hotelling model, Secure strategy equilibrium, Spatial competition*

Abstract: *The problem of spatial competition was formulated in 1929 by Harold Hotelling. He considered two firms playing a two-stage game. They choose locations in stage 1 and prices in stage 2. If locations are chosen by competitors Nash equilibrium do not always exist. For studying these cases we employ the concept of the secure strategy equilibrium (SSE) which allows to solve the game of choosing prices for any locations. We examine a nontrivial particular case when prices grow if market moves from a monopoly to a duopoly.*

We study the classical problem of the effect of competition on locational decisions proposed by H.Hotelling in 1929 (*The Economic Journal*, 93, 153, (1929)). He considered two firms selling the same product and playing a two-stage game: in the first stage each firm chooses location of its sell point; in the second stage, the firm defines a price for its product. Consumers are spread equally along the interval and their choice is defined by prices and the travel costs proportional to the distance.

One considers an interval [A,B] of length l which can be interpreted as a street in a city, beech line, highway or some horizontally differentiated product in the model of monopolistic competition. Consumers are identical and uniformly spread on the interval with a particular density which can be taken as 1 without loss of generality. The sell points of players 1 and 2 are located at the positions a and $l-b \geq a$ respectively.The distance between them is $d = l - a - b \geq 0$. Each consumer pays some price for traveling for a unit distance, which can also be taken as 1 without loss of generality. Consumer at a given point buys the unit of product only if the sum of its price and the cost of travel to the sell point is less than the unit utility. Consumers have no preferences in choosing the firm other than minimizing the sum of the product price and travel cost. Therefore the amount of the product sold by each player q_1 and q_2 equals the length of the line segment with the consumers who choose this player.

One seeks for the subgame perfect equilibrium in the two-stage dynamical game:

Stage 1. Sellers choose their locations a and b ($a + b < l$).

Stage 2. Sellers define prices for their product p_1 and p_2.

The profit function of the first player is:

$$u_1(a, b, p_1, p_2) = p_1 q_1 = \begin{cases} p_1(\nu_1 + w_1), & \text{if } p_1 < p_2 - \delta; \\ p_1(\nu_1 + r_1), & \text{if } |p_1 - p_2| \le \delta; \\ 0, & \text{if } p_1 < p_2 + \delta \end{cases}$$

where $\nu_1 = \min\{a, 1 - p_1\}$, $w_1 = \min\{\delta + b, 1 - p_1\}$, $r_1 = \min\left\{\dfrac{\delta + p_2 - p_1}{2}, 1 - p_1\right\}$

The profit function of the second player is symmetrical.

The principal problem is that under Hotelling's assumptions the price-setting subgames posses equilibria in pure strategies for only a limited set of location pairs. D'Aspremont et al. (*Econometrica*, 47, 5, (1979)) defined conditions under which the price equilibrium in Hotelling's model fails to exist. Most of subsequent papers can be divided into three groups. 1) Modification of the travel cost function in such a way which ensures the Nash equilibria existence at all locations (D'Aspremont et al. 1979). 2) Solving Hotelling problem in the mixed strategies (Osborne M.J. and Pitchik C., *Econometrica*, 55, 4,1987)). 3) More sophisticated modifications of Hotelling model which for example take into account the preferences of consumers (Ahlin P., *Duke Journal of Economics*, IX, 1997).

Our aim is to solve original Hotelling problem in terms of the equilibrium in secure strategies (ESS). The principle of ESS was proposed in (Iskakov M.B., *Automation and Remote Control*, 66, 3, (2005) and 69, 2, (2008)) as the generalization of Nash equilibrium. It coincides with the Nash Equilibrium when Nash Equilibrium exists and takes into account the intention of players to maximize their profit under the condition of security against the actions of other players. This approach reflects the natural logic of behavior of players in this model. We demonstrate that the ESS in the price-setting subgames exists for all location pairs in the original Hotelling model and provide symmetrical solution for the two-stage game.

Below we provide principal definitions of ESS for the game $G = (S_i, u_i, i \in N)$.

Definition 1. The threat of player j to player i is the pair of strategy profiles $\{s,(s'_j,s_{-j})\}$ such that $u_j(s'_j,s_{-j}) > u_j(s)$ and $u_i(s'_j,s_{-j}) < u_i(s)$. The profile s is said to contain the threat to player i. The profile (s'_j,s_{-j}) and the strategy s'_j of player j is said to threaten to player i.

Definition 2. Strategy s_i of player i is a simple secure strategy at a given s_{-i} if the profile s does not contain any threats for player i.

Definition 3. The set $W_i(s)$ of preferable strategies secured against threats is the set of strategies s'_i of player i at a given s such that $u_i(s'_i,s_{-i}) \geq u_i(s)$ and provided that $u_i(s'_i,s'_j,s_{-ij}) \geq u_i(s)$ for any threat $\{(s'_i,s_{-i}),(s'_i,s'_j,s_{-ij})\}$ of player $j \neq i$ to player i.

Definition 4. The profile s^* is a simple Equilibrium in Secure Strategies (ESS) if and only if for all i we have that

$$s_i^* \in \arg\max_{s_i \in W_i(s^*)} u_i(s_i,s_{-i}^*).$$

The symmetrical solution of the two-stage Hoteling game in secure strategies is given in the table below:

l	$l \leq 0.8$	$l \in [0.8,8/7]$	$l \in [8/7,4/3]$	$l \in [4/3,2]$	$l \in [4/3,2]$
a,b	$l/4$	$\frac{2l-1}{3}$	$1-\frac{l}{2}$	$\frac{l}{4}$	$a,b \geq 0.5, a+b \leq l-1$
$p*_{1,2}$	l	$\frac{4-2l}{3}$	$\frac{l}{2}$	$1-\frac{l}{4}$	0.5
$u*_{1,2}$	$\frac{l^2}{2}$	$\frac{(2-l)l}{3}$	$\frac{l^2}{4}$	$\frac{(4-l)l}{8}$	0.5

n-Person Stochastic Games of "The Showcase Showdown"

Anna Ivashko[1] and **Evgeny Ivashko**[2]

Institute of Applied Mathematical Research
of Karelian Research Center of RAS
Russia
[1]*afalco@krc.karelia.ru*
[2]*ivashko@krc.karelia.ru*

Keywords: *Dynamic Programming approach, Threshold strategy, n-person game*

Abstract: *The following game is considered. Each of n players independently observes one or two random variables. Getting the first observation the player have to decide either to stop or to continue for a second draw. No information about observations or decisions of the other players is available. The object of each player is to get the highest total score without going over 1. In the case the total scores of all players exceed 1, wins the player whose score is closest to 1. The regular and the disorder versions of the game are investigated. We propose the formula for optimal threshold strategy which maximizes the probability of winning in the n-person game. Numerical results are given.*

Kaynar [2] considered the version of the Showcase Showdown game of chance that played on every TV game show "The Price is Right". The game is as following. Each of n players independently observes one or two random variables. Getting the first observation the player have to decide either to stop or to continue for a second draw. No information about observations or decisions of the other players is available. The object of each player is to get the highest total score without going over 1. In the case the total scores of all players exceed 1, wins the player whose score is closest to 1.

To find out the optimal thresholds Kaynar enumerated all the winning probabilities and gave the final formulas for two- and three-person games. But the same approach couldn't be used to find out the optimal strategy for the game with arbitrary number of players.

We use the dynamic programming approach to construct the final formula allowing to find the optimal thresholds for n-person game. Finally the solution of the following equation gives the optimal threshold t maximizing the probability of the winning in the n-person game:

$$t^{2(n-1)} = \frac{1-t^{2n}}{n(t+1)} + \frac{t^{2n-1}}{2^{n-1}(2n-1)}.$$

For the cases of two and three players it gives the same result as Kaynar got.

Additionally we considered the disorder-versions of the game. We propose the formula for optimal threshold strategy which maximizes the probability of winning in the n-person game. Numerical results are also given.

Some versions of the game of chance also have been analyzed by Coe and Butterworth [1] and Tijms [6]. The dynamic programming approach for optimal stopping problem have been considered in [3,4,5].

The research was supported by the Division of Mathematical Sciences (the program "Mathematical and algorithmic problems of new information systems").

The work is supported by Russian Foundation for Basic Research, project 10-01-00089-a.

References

[1] Coe, P.R. and Butterworth, W. (1995). Optimal stopping in "The Showcase Showdown", *The American Statistician* 49: 271-275.

[2] Kaynar, B. (2009). Optimal stopping in a stochastic game, *Probability in the Engeneering and Information Sciences* 23: 51-60.

[3] Mazalov, V. V. (1996). A Game related to optimal stopping of two sequences of independent random variables having different distributions, *Mathematica Japonica* 43, N 1: 121-128.

[4] Sakaguchi, M. (2005). Equilibrium in two-player games of "Showcase Showdown", *Scientiae Mathematicae Japonicae* 61, 145-151.

[5] Sakaguchi, M. (2007). Players' information in two-player games of "Score Showdown", *Game Theory and Applications* 11, Edit. Petrosjan and Mazalov, N.Y., Nova Science Publ.: 111-124.

[6] Tijms, H.C. (2007). *Understanding probability*, 2nd ed. Cambridge: Cambridge Univ. Press.

Optimal Strategies in the Best-choice Problem with Disorder*

Evgeny E. Ivashko

Institute of Applied Mathematical Research,
Karelian Research Centre RAS
ivashko@krc.karelia.ru

Keywords: *Best-choice, Disorder, Full-information best-choice problem*

Abstract: *The following version of the full-information best-choice problem with disorder is considered. A production system is working in the state S_1 and there is the constant probability α that it falls into the state S_2. The system produces the iid random variables that distributed by the laws $F_1(x)$ and $F_2(x)$ in the states S_1 and S_2 respectively. The observer knows parameters $F_1(x)$, $F_2(x)$ and α but the real state of the system is unknown. The aim of the observer is to maximize the probability of choosing the largest value from the finite random sequence. We find the optimal rule in the class of strategies where observer doesn't estimate the real state of the system. The numerical results are also given.*

The following version of the full-information best-choice problem with disorder is considered. A production system is working in the state S_1. In this state the system produces independent identical distributed random variables (i.i.d. r.v.) X_k^1, $k = 1,2,...$ with the absolutely continuous cumulative distribution function (cdf) $F_1(x)$. In the random time θ ($\theta = 1,2,...$) the system falls into the state S_2 (the disorder happens). In the state S_2 the system produces i.i.d. r.v. X_k^2, $k = 1,2,...$ with the absolutely continuous cdf $F_2(x)$. The disorder moment θ has a geometric distribution with the parameter $\alpha (0 \le \alpha \le 1)$, i.e. $P(\theta = j) = \alpha^{j-1}(1-\alpha)$.

At each time k, $k = 1,2,...$ the decision-maker observes the r.v. $X_k = X_k^1 I_{\{\theta > k\}} + X_k^2 I_{\{\theta \le k\}}$ and have to choose either to STOP (and accept the value X_k) or CONTINUE (reject the X_k and observe X_{k+1}). The decision-maker knows the cdfs $F_1(x)$ and $F_2(x)$ and the parameter α but the real state of the system is unknown. The

* The work is supported by Russian Found for Basic Research, project 10-01-00089-a.

object is to maximize the probability of choosing the largest value from the random sequence $\{X_k\}_{k=1}^{n}$. The time horizon is n: if the observer rejected $n-1$ observations, she have to stop at n -th. The observation once rejected cannot be recalled later.

We find the optimal rule in the class of strategies where observer doesn't estimate the state of the system. The numerical results are also given.

This problem is the generalization of the full-information best-choice problem considered in [1] or [2] to the case of the disorder. The other versions of the best-choice problem with disorder are investigated in [3,4].

References

[1] B. A. Berezovsky and A. V. Gnedin. The Best Choice Problem. Nauka, Moscow. 1984.

[2] Gilbert J. P., Mosteller F. Recognizing the maximum of a sequence , Journal of the American Statistical Assosiation, Vol. 61, No. 313, 1966, pp. 35-73.

[3] V. Mazalov, E. Ivashko. Best-choice problem with disorder , Surveys in Applied and Industrial Mathematics, V. 14, Iss. 2, 2007, pp. 215-224 (in russian).

[4] M. Sakaguchi. A best-choice problem for a production system which deteriorates at a disorder time , Scientiae Mathematicae Japonicae Vol. 54, No. 1, pp. 125-134.

The Shapley Value for Games with Restricted Cooperation

Ilya Katsev

St. Petersburg Institute for Economics and Mathematics,
Russian Academy of Sciences,
Tchaikovsky st. 1, 191187 St. Petersburg.
katsev@yandex.ru

Keywords: *Cooperative game, Shapley value, Restricted cooperation, Balanced contribution*

Abstract: *This paper deals with games with restricted cooperation. In a case of restricted cooperation only some coalitions (feasible coalitions) can cooperate. Three solutions for games with restricted cooperation will be presented. One of them (Myerson value) in well known. Two others based on the same principle: to construct some restricted game and use the Shapley value for this game. An axiomatic characterization for each possible solution which can be constructed by this way will be given. .*

Introduction

The cooperative game theory usually deals with problems of a standart type: as a start point we have a set of players and a characteristic function (which shows how much profit each coalition can get in a situation of cooperation) and as a result we get a "fair" distribution of the profit between players.

However, in many real life situations not every group of players has the opportunity to cooperate and to collect their own payoff. We say that we deal with cooperative games with restricted cooperation when not all coalitions can form. The reason of restrictions on the collection of feasible coalitions can be various, for instance restrictions induced by law, restrictions on the maximum number of players that are allowed to cooperate, restrictions because their is no full communication between players or restrictions because players need consent of their superiors to form coalitions with others.

A situation which can be described by the player set N and the characteristic function $v: 2^N \to \mathbb{R}$ we call a classical cooperative game and a more general situation which can be described by triple (N, v, Ω) where $\Omega \subset 2^N$ is a collection of *feasible coalitions* we call a *game with restricted cooperation*. The collection Ω consists of coalitions with the property that players of such coalition can cooperate together. The collection of all games with the restricted cooperation is denoted by GR.

So finding of the fair distribution of the payoff for games with restricted cooperation is a more general problem that finding of this distribution for classic cooperative games. We will consider the generalization of the Shapley value for games with restricted cooperation.

In this paper we use the next properties of solutions:

Property 1 (Non-cooperative player property) *A solution x has non-cooperative player property if for every game with restricted cooperation (N,v,Ω) it holds that if $i \in N \setminus \bigcup_{S\in\Omega} S$ then $x_i(N,v,\Omega) = 0$.*

and just stronger

Property 2 (Non-cooperative player out property) *A solution x has non-cooperative out player property if for every game with restricted cooperation (N,v,Ω) it holds that if $i \in N \setminus \bigcup_{S\in\Omega} S$ then*

1. $x_i(N,v,\Omega) = 0$

2. $x_j(N \setminus \{i\}, v_{N\setminus\{i\}}, \Omega) = x_j(N,v,\Omega)$ *for each* $j \in N \setminus \{i\}$.

Property 3 (Weak efficiency) *A solution x is weak efficient if for every game with restricted cooperation (N,v,Ω) with $N \in \Omega$ it is true that*

$$x(N) = v(N)$$

Examples of values

Three values are defined in this section by the same way. For each of these values in the first we define a restricted game and after we consider the Shapley value of the restricted game. So the differences between three solutions are only in the ways to define the restricted game.

For the Myerson value the restricted characteristic function (N,v,Ω) is defined as $r(S) = \sum v(S_i)$ where S_i are components of S. For the union value $r(S) = v(\sigma(S))$ where $\sigma(S)$ equal to union of all feasible subsets of S. Finally, for the max-value $r(S) = \max_{T\subset S, T\in\Omega} v(S)$.

About a history of these three values:

Myerson (1980) defined and characterized the Myerson value for arbitrary collection of feasible coalitions with the only condition that all singletons are feasible. In this paper we consider the generalization of this value for the case of arbitrary set system (so we can delete the condition about a feasibility of singletons).

The union value was defined and characterized in van den Brink, Katsev, van der Laan (2009) for situations where collection Ω is union closed. In this paper we define this value for situation with an arbitrary set system Ω.

The max-value value is defined in this paper.

The max-value has the next properties: fairness, balanced contribution, non-cooperative player out property and weak efficiency. But it don't have additivity. Instead of it the value has two another important properties:

Property 4 (Independence of irrelevant coalitions) *A value x is independent of irrelevant coalitions is it doesn't change with changing of the characteristic function v on non-feasible coalitions.*

Property 5 (Monotone positiveness) *A value x for games with restricted cooperation has a monotone positiveness property if for every monotone and non-negative game (N,v) ($v(S) \geq 0$ for all $S \subset \Omega$) it is true that the value x is also non-negative ($x_i(N,v,\Omega) \geq 0$ for all $i \in N$).*

It is simple to see that both Myerson and union values have not all of properties of the max-value. Moreover, it is possible to prove the next theorem:

Theorem 2.1 There is no value for games with restricted cooperation with the next properties: additivity, fairness, non-cooperative player out property, independence of irrelevant coalitions, weak efficiency and monotone positiveness.

This theorem shows that the considering of max-value has sense and we cannot find any solution with all properties of this value plus additivity.

The general axiomatization

Let us consider a class Ω of solutions for games with restricted cooperation. A solution belongs to this class if it has next properties: balanced contribution and non-cooperative player property.

Let us formulate the main result of this paper:

Theorem 3.1 *For every pair of different solutions $x, y \in A$ there is a game with restricted cooperation (N,v,Ω) such that $x(N) \neq y(N)$.*

The theorem says that if we know that a solution x has balanced contribution property and non-cooperative player property then it is sufficient to know the value $x(N)$ for each situation (N, v, Ω) for finding x on the class of games with restricted cooperation. For a characterization of 3 solution which was defined in the previous section we should give three different definitions of efficiency.

Definition 3.2 *A solution x is 'Myerson efficient' if for every ^{2} we have*

$$x(N) = \sum_{C \in C_{\Omega}(N)} v(C)$$

Definition 3.3 *A solution x is 'union efficient' if for every (N, v, Ω) we have*

$$x(N) = v\left(\bigcup_{S \in \Omega} S\right)$$

Definition 3.4 1 *A solution x is 'max-efficient' if for every (N, v, Ω) we have*

$$x(N) = \max_{S \in \Omega} v(S)$$

It should be noted that each of approaches looks reasonable. We can formulate an evident corollary of the theorem which gives us three characterization of values defined above:

Corollary 3.52 *1. The Myerson value is the unique value with balanced contribution, non-cooperative player property and Myerson efficiency*

2. The union value is the unique value with balanced contribution, non-cooperative player property and union efficiency

3. The max-value is the unique value with balanced contribution, non-cooperative player property and max-efficiency

Let us give one more theorem which says that there is a one-to-one correspondence between kinds of efficiency and solutions for games with restricted cooperation with balanced contribution property.

Theorem 3.6 3 *Consider a function $f : GR \to \mathbb{R}$. Let us suppose that f has the next property: if $\Omega = \varnothing$ then $f(N, v, \Omega) = 0$.*

Then there is a unique single-valued solution for games with restricted cooperation $x \in A$ such that for each game $(N, v, \Omega) \in GR$ it holds that

$$\sum_{i \in N} x_i(N, v, \Omega) = f(N, v, \Omega)$$

The Restricted Prenucleolus

Ilya Katsev[1] and **Elena Yanovskaya**[2]

St.Petersburg Institute for Economivs and Mathematics
Russian Academy of Sciences
Russia
[1]*katsev@yandex.ru*
[2]*eyanov@emi.nw.ru*

Keywords: *Cooperative game, Restricted cooperation, Prenucleolus*

Abstract: *A game with restricted cooperation is a triple (N,v,K), where N is a finite set of players, K is a collection of feasible coalitions of players including the grand coalition N, and v is a characteristic function defined on the collection K. The definition implies that if K consists of all coalitions, then the game (N,v,K)=(N,v) is the classical cooperative game with transferable untilities (TU). The class G of all games with restricted cooperation with an arbitrary universe of players is considered. A solution PN_r called the restricted prenucleolus is defined for the class G by the same way as the prenucleolus is defined for classical TU games. For every game (N,v,K) the solution* $PN^r(N,v,\Omega)$ *consists of all payoff vectors which lexicographically minimize ordered excess vectors for the coalitions from K. Necessary and sufficient conditions for existence of the restricted prenucleolus and those providing its single-valuedness are given.*

A game with restricted cooperation is a triple (N,v,Ω), where N is a finite set of players, $\Omega \subset 2^N$, where $N \in \Omega$, is a collection of feasible coalitions, $v:\Omega \to \mathbb{R}$ is a characteristic function. The definition implies that if $\Omega = 2^N$, then the game $(N,v,\Omega) = (N,v)$ is a classical TU cooperative game.

The restricted prenucleolus for a class of games with restricted cooperation is defined as a solution, consisting for each game (N,v,Ω) of the set of its payoff vectors which lexicographically minimize ordered excess vectors for the coalitions from Ω.

For "poor" collections Ω it may be set-valued and even may not exist. For example, for $N = \{1,2,3\}$, $\Omega = \{1,2\},\{2,3\}$ for any characteristic function v the restricted prenucleolus does not exist, because the minimum of $\max\{e(\{1,2\},x), e(\{2,3\},x)\}$ over efficient payoff vectors is not attained since this maximum tends to $-\infty$ when $x_2 \to \infty$.

Note that existence or not existence of the restricted prenucleolus depends only on Ω, but not on a characteristic function v.

Let N be fixed, Ω be a collection of coalitions of N such that the restricted prenucleolus $PN^r(N,v,\Omega)$ exists for all characteristic functions v. Let $x \in PN^r(N,v,\Omega)$, $\mathbf{S}_1(x,v,\Omega) \subset \Omega$ be the collection of coalitions on which maximal excesses for x are attained:

$$\mathbf{S}_1(x,v,\Omega) = \{S \in \Omega | v(S) - x(S) = \max_{Q\in\Omega}\{v(Q) - x(Q)\}.$$

Lemma 1. *The collection* $\mathbf{S}_1(x,v,\Omega)$ *is either balanced or equals* Ω *and* Ω *is weakly balanced.*

For an arbitrary collection of coalitions $\mathbf{S}$ consider $|\mathbf{S}| \times |N|$ matrix $||\mathbf{S}|| = ||\chi_S||, S \in \mathbf{S}$. We say that the collection $\mathbf{S}$ has *rank m*, if the rank of matrix $||\mathbf{S}||$ is equal to *m*.

Theorem 1 *Given an arbitrary game* (N,v,Ω), *its restricted prenucleolus* $PN^r(N,v,\Omega)$ *is single-valued if and only if the collection* Ω *is balanced and has rank* $n = |N|$.

Let G_b^r be the class of games with restricted cooperation defined by balanced and having the full rank collections of feasible coalitions.

Proposition 1 *The restricted prenucleolus for the class* G_b^r *is efficient, anonymous w.r.t. the players and the coalitions, covariant, and consistent.*

Single-valuedness and anonymity of the restricted prenucleolus imply its *equal treatment property* defined by

$$v(S \cup \{i\}) = v(S \cup \{j\}) \text{ for all } S \in \Omega, i, j \notin S \Rightarrow PN_i^r(N,v,\Omega) = PN_j^r(N,v,\Omega).$$

The class G_b^r as a class of games with the single-valued restricted prenucleolus can be widen by extending the equal treatment property to another property of symmetry of a solution.

Definition 1 *Two players* $i, j \in N$ *are identical in* $\Omega \subset 2^N$ *if for each* $S \in \Omega$ *either* $i, j \in S$, *or* $\{i, j\} \cap S = \varnothing$.

Evidently, in classical TU games with $\Omega = 2^N$ there are no identical players in the definition. When $\Omega \subset 2^N$, $\Omega \neq 2^N$ identical players may exist. Such players have identical powers in all games with the feasible collection Ω independently of their characteristic functions.

Definition 2 Given $N, v(N)$ and a collection Ω of coalitions from N, the *set of egalitarian payoff vectors* is the set

$$S(N,v,\Omega) = \{x \in X(N,v,\Omega) | x_i = x_j \text{ if } i,j \text{ are identical in } \Omega\}.$$

Definition 3 A *symmetric restricted prenucleolus* (PN^{rs}) for the game (N,v,Ω) is the set of egalitarian payoff vectors defined by

$$PN^{rs}(N,v,\Omega) = \{x \in PN^{r}(N,v,\Omega) | x_i = x_j \text{ if } i,j \text{ are identical in } \Omega\}.$$

In view of this definition it may turn out that the restricted prenucleolus for some game is set-valued, but the symmetric restricted prenucleolus is single-valued. Let us give the conditions providing single-valuedness of the PN^{rs}.

Theorem 2 $| PN^{rs}(N,v,\Omega) | = 1$ *if and only if the collection* Ω *is balanced and has rank* $k(\Omega)$, *where* $k(\Omega)$ *is the number of classes of identical players*.

In cooperative game theory there are models named *cooperative games with coalitional structures* (CGCS). Such a model is a triple $(N, v, \mathbf{B})$ where N is a set of players, $v : 2^N \to \mathbb{R}$ is a characteristic function, $\mathbf{B} = (B_1,...,B_k)$ is a partition of N. Solutions for CGCS are defined as those for classical cooperative games with taking into account the partitions as the first level of cooperation. The most known value for CGCS is the Owen value (Owen 1977), and the "Shapley-Shapley" value (Kamijo 2009) both being modifications of the Shapley value. Note that the collections of feasible coalitions for these values differ: for the Owen value the collection Ω consists of 1) all unions of the coalitions of the partition, 2) all subsets of each coalition of the partition, and 3) unions of one subset of a coalition and arbitrary coalitions of the partition. The Shapley-Shapley value depends only on coalitions of 1) and 2) types.

In the definition of these solutions it is possible to replace the Shapley value by any other TU game value. In all cases we obtain new values for CGCS. We replace the Shapley value by the prenucleolus and obtain two new solutions: the *Owen-type prenucleolus* and the *Kamijo-type prenucleolus*. Axiomatizations of both solutions have been obtained.

Theorem 3 *For the class of CGCS with an infinite universe set of players the Owen-type prenucleolus is the unique value satisfying the properties: non-emptiness, covariance, outer symmetry, inner symmetry, coalitional consistency, consistency games with only one coalition in a partition.*

The new property - coalitional consistency - means that only coalitions of the partition may leave the game.

Theorem 4 *For an infinite universal set of players the Kamijo-type prenucleolus is the unique value for the class of CGCS satisfying the properties: non-emptiness, covariance, anonymity w.r.t. the players, and consistency.*

Acknowlegments. Research for this paper was supported by the Russian Foundation for Basic Research, grant N 09-06-00155a.

References

[1] Owen, G. (1977). Values of games with a priori unions. In: Essays in Mathematical Economics and Game Theory (eds. Henn R. and Moeschlin O.), Springer-Verlag, Berlin, 76 –88.
[2] Kamijo, Y. (2009). A two-step Shapley value for cooperative games with a coalition structures. International Game Theory Review, 11, N2, 207-214.

ВЕСТНИК САНКТ–ПЕТЕРБУРГСКОГО УНИВЕРСИТЕТА

Серия 8
МЕНЕДЖМЕНТ

Ответственный редактор серии:
Ю.Е. Благов

Издательство Высшей школы менеджмента Санкт-Петербургского университета

Values for Cycle-free Directed Graph Games

Anna Khmelnitskaya[1] and **Dolf Talman**[2]

[1]*Institute for Economics and Mathematics RAS,*
Saint Petersburg, Russia
a.khmelnitskaya@math.utwente.nl
[2]*Tilburg University,*
Netherlands

Keywords: *TU game, Cooperation structure, Cycle-free directed graph, Sharing a river*

In standard cooperative game theory it is assumed that any coalition of players may form. However, in many practical situations the collection of feasible coalitions is restricted by some social, economical, hierarchical, communicational, or technical structure. The study of TU games with limited cooperation introduced by means of communication graphs was initiated by Myerson (1977). In this paper we restrict our consideration to the class of cycle-free directed graph games in which all players are partially ordered and a possible communication via bilateral agreements between participants is presented by a directed graph (digraph) without directed cycles. A cycle-free digraph cooperation structure allows modeling of various flow situations when some links may merge while others split into several separate ones. Following Myerson, we assume that for a given game with cooperation structure, cooperation is possible only among productive coalitions of players, while in a directed graph not every connected coalition might be productive.

We introduce values for cycle-free digraph games axiomatically and provide their explicit formula representation, we also study their stability. Furthermore, we show that the problem of sharing a river with a delta and with multiple sources among different agents located at different levels along the river bed can be embedded into the framework of a cycle-free digraph game.

Analysis of a Two-Person Positional Nonzero-Sum Differential Game with Integral Payoffs

Anatolii F. Kleimenov

Institute of Mathematics and Mechanics
Ural Brunch of RAS
Russia
kleimenov@imm.uran.ru

Keywords: *Non-antagonistic positional differential game, Integral payoffs, Vector criterion, Acceptable trajectories*

Abstract: *A two-person non-antagonistic positional differential game with simple dynamics and with cost functionals of players containing integral terms is considered. Besides, one of players has two goals (vector criterion).The formalization of players' strategies in the game is based on the formalization and results of antagonistic positional differential game theory developed by Krasovskii and Subbotin. The analysis of the game uses also results of the non-antagonistic positional differential game theory obtained by the author. There are several stages in research of the problem. At the beginning, the set of attainability for given initial condition and given constraints is constructed. Then, so-called acceptable trajectories are selected. Thereafter, we separate out Pareto optimal trajectories, which are acceptable for both players. Finally, optimal strategies are constructed.*

Consider a two-person non-antagonistic positional differential game (NPDG) which dynamics is described by the following equation

$$\dot{x} = u + v, \ \ x, u, v \in R^2 \ , \ \|u\| \le \mu, \ \ \|v\| \le \mu \ , \ x(t_0) = x_0 \ .$$

Cost functionals of the first player (P1) I_1 and of the second player (P2) I_2 are given

$$I_1 = \|x(\vartheta)\| + \int_{t_0}^{\vartheta} w(\tau)\|u(\tau) + v(\tau)\|^2 \, d\tau \to \min,$$

$$I_2 = (-\|x(\vartheta) - a\|, \int_{t_0}^{\vartheta} w(\tau)\|u(\tau) + v(\tau)\|^2 \, d\tau) \to \max \ ),$$

where a is a given point in the plane; ϑ is the fixed final time of the game.

Control u is governed by P1, while P2 governs a pair of controls (v, w), where $\alpha_1 \le w \le \alpha_2$.

One can interpret the problem in the following way. P1, who chooses his control u, tends to minimize the distance from the origin at the moment ϑ, and simultaneously to minimize the total "energy" cost under condition that energy "price" w is chosen by P2. In turn, P2, who chooses his control v and energy "price" w, has two goals (vector criterion): the first one is to minimize the distance from the aim point a at the moment ϑ, and the second one is to maximize the total energy cost.

Assume that both players know value of the phase vector $x(t)$ of the game at the current moment of time t. Besides, assume that P1 knows values of both controls of P2 at the current moment of time t. The formalization of players' strategies in the game is based on the formalization and results of antagonistic positional differential game theory [1,2]. The analysis of the game uses also results of the non-antagonistic positional differential game theory [3,4].

A particular case of the problem is when, firstly, $w(t) \equiv const$, and, secondly, P2 has only one goal, notably, second one, was considered in [3].

There are several stages in research of the problem. At the beginning, the set of attainability for given initial condition and given constraints is constructed. Then, so-called acceptable trajectories are selected. Thereafter, we separate out Pareto optimal trajectories, which are acceptable for both players. Finally, optimal strategies are constructed.

The report is supported by Russian Foundation of Basic Research, grant 09–01–00313.

References

[1] Krasovskii, N.N. (1985). Control of a Dynamical System. Nauka. Moscow (in Russian).
[2] Krasovskii, N.N. and A.I. Subbotin (1988). Game-Theoretical Control Problems. Springer. NY. Berlin.
[3] Kleimenov A.F. (1993). Nonantagonistic Positional Differential Games. Nauka. Ekaterinburg (in Russian).
[4] Kleimenov,A.F. (2008). Solutions in non-antagonistic positional differential games with vector payoff functionals. // DVD of Preprints of the 17th IFAC World Congress, July 6-11, 2008, Seoul, Korea.

Graph Structures and Algorithms in Multidimensional Screening

Sergey Kokovin[1], **Babu Nahata**[2] and **Evgeny Zhelobodko**[3]

[1,3]*Department of Economics, Novosibirsk State University,*
Novosibirsk 630090, Russia,
skokovin@math.nsc.ru, ezhel@ieie.nsc.ru[*]
[2]*Department of Economics University of Louisville,*
Louisville, Kentucky 40292, USA
nahata@louisville.edu

Keywords: *Multidimensional screening, Second-degree price discrimination, Graph structures*

JEL Codes: D42, D82, L10, L12, L40.

This paper developes the graph-theory approach to multidimensional screening. It describes the graph-structures of solutions, their characterizationg and method of finding them through graphs, for revealing their properties and comparative statics. Our setting considers discrete consumer types, one outside option –non-participation, and one pricing tool –tariff for a quantity/quality bundle. Most results require only quasi-linearity of utilities and separability of costs w.r.t. bundles.

A solution structure is a list of active incentive-compatibility and participation constraints percepted as a directed arc ($i \to j$) connecting two agent-nodes i, j . This " A-graph" is a " river"when it is in-rooted and dicycles (closed directed paths) are absent. To overcome dicycles and other technical hardships, this paper applies a small parameter $\rho \geq 0$ relaxing each constraint.

Under quite weak assumptions on preferences and costs we find that any solution has an in-rooted A-graph, and under strict relaxation $\rho > 0$ any A-graph is a river, while bunching among predecesors and successors in the graph is excluded. Thus, two major impediments in characterizing and finding solutions are overcome: (1) unclear

[*] We gratefully acknowledge the research support from the University of Louisville, the Economic Education and Research Consortium (EERC), financed by the Eurasia Foundation; USAID; GDN; the World Bank Institute and the Government of Sweden (EERC grant R06-0561). We are grateful for the assistance and comments from Richard Ericson, Victor Polterovich and especially Alexei Savvateev.

existence of the Lagrange multipliers; (2) often more narrow list of positive multipliers than the list of active constraints.

More surprizingly, the reverse is true: every river is a solution structure for some screening problem. We enumerate all possible rivers, i.e., different " regimes" of screening: 5 for two consumers, 79 for three consumers, 2865 for four, and so on.

Based on these findings, we characterizes any solution through its spanning-tree without first-order conditions (FOC) and, under differentiability and relaxation $\rho > 0$, through FOC. Therefore, to find all solutions in (generally non-convex) optimization it is sufficient to try FOC for all rivers and then compare the resulting local optima.

Related " trees-and-rivers" algorithm first classify the variety of solution structures (trees and rivers) according to longest-path preorder. Each preorder-family is explored starting with its spanning-tree. If the tree-specific solution to the optimization program turns out feasible (i.e., the unconstrained argmaximum of the profit function satisfies all inequalities), then studying the rest of this family becomes redundant. Otherwise we obtain the upper bound on profit attainable from this family. The whole family is rejected when this new family-specific upper bound is lower than any feasible lower bound obtained previously. Further, if the plan is found incompatible with any not-in-tree inequality but the family is not rejected, the tree-specific program is enriched by additional constraints starting with the violated one. Adding these " bypasses" transforms the tree-specific program into the river-specific program. Through supplementing the tree with all its downstream bypasses, all rivers in this family can be explored. But only some should be explored, since each trial updates the upper and lower bounds on profit for rejecting some families and sub-families of rivers without trying. Exhausting the list of all possible spanning-trees and related families of rivers completes the algorithm.

We design also another, "relaxation" algorithm for finding the screening optima. It starts optimization with the first-best non-feasible solution, i.e., solution when all incentive compatibility constraints are relaxed for big $\rho = \infty$. Then the relaxation is gradually decreased to $\rho = 0$ through certain steps. At each step one or several local optima emerge, their set is explored to find the true optimum.

The comparison of the two algorithms in efficiency remains an open question.

Stochastic Cooperative Games as an Instrument for Modeling of Relations of Public Private Partnership

Pavel Konyukhovsky[1] and **Margarita Solovyeva**[2]

Saint-Petersburg University,
Russia
[1]*p.konyukhovsky@econ.pu.ru*
[2]*mapro@list.ru*

Keywords: *Stochastic cooperative games, Public private partnership*

Abstract: *Problems concerning application of cooperative game theory's tooling under the simulation of relationships of public private partnership (PPP) are considered in the thesis. Special focus is devoted to the situation of non-deterministic utilities received by potential coalitions of participants. Under the terms of named situation an approach has been offered based on the concept of stochastic cooperative games.*

Under the conditions of modern Russian economy public private partnership (PPP) is becomming more and more popular and effective instrument for complex solving of infrastructure problems, that from one side are in the responsibility's zone of the government, and from the other side are the tasks of entrepreneurial business of the firms. The importance of PPP's role has strengthened at the stage of negotiation of Russian economy the consequences of world financial crisis.

The problem of income's distribution between participants of PPP project is among the topest-priority tasks arose under investigation of cooperation of participants that implement project by joining forces of government and private sector. Its formulation predefines to a considerable degree the choice of hardware tools that are adequate the environment of so complicate and diversified object as PPP used to be. Therefore the idea of use of cooperative game theory seems to be highly productive and attractive. In the simplest case the process of alliance of potential investors in partnerships can be presented in the form of a classical cooperative game consisted of n players with transferable utility. This model opens quite broad possibilities for analysis concerning relationships of uprising and evolution of PPP's associations on the base of classical approaches to solving cooperative games. Primarily it's C-core, N-core, K-core and vector Shapley. However it can not be denied that one of the crucial problems among others that are emerged under implementation of game-theoretic approaches in modeling PPP becomes the problem of construction of characteristic function in a cooperative game. In particular admission, concerning possibility of describing of potential gains for each potential coalition, consisted of supposed participants of PPP's projects with determinate values, seems to be quite disputable. Considerably supposal of stochastic character of these values appears to be more realistic and attractive. However under these assumptions we lost the possibility of direct introduction of classical cooperative game's tooling. The pass to tools of stochastic cooperative games becomes one of the possible methods for overcoming of this difficulty.

A setup consisted of set of players $I = \{1..m\}$ and characteristic function $\tilde{v}(S)$ assigning to any subset (coalition) of players $S \subset I$ one gain attained by this coalition will be taken as stochastic cooperative game (SCG) with transferable utility. The principal distinction of suggested definition of stochastic cooperative game from the traditional definition of classical cooperative games with transferable utility lies in the fact that in given definition gain of coalition $\tilde{v}(S)$ is supposed to be a random variable with any known distribution function $F_S(x)$. In the following stochastic cooperative game will be briefly denoted by $(I, \tilde{v})$. .

A vector $x \in R^n$ given distribution of utilities between participants of a game can be considered as an imputation in SCG. This vector fulfills the following conditions:

(1) individual rationality of players

$\mathbf{P}\{x_i \geq \tilde{v}_i\} \geq \alpha$ or what is the same $x_i \geq F_i^{-1}(\alpha)$;

(2) reachability of gain provided a imputation for a full (gross) coalition

$$\mathbf{P}\{\sum_i x_i \leq \tilde{v}(I)\} \geq \alpha \text{ or } \sum_i x_i \leq F_I^{-1}(\alpha),$$

where $F_i^{-1}(\alpha)$ is a fractile for probability α of distribution functions for a player i ($F_I^{-1}(\alpha)$ - for a full coalition, respectively)

Under given approach to cooperative games such concepts as superadditivity and convexity can receive interesting development.

Particularly a game will be named strongly superadditivity if for any α

$$F_{S \cup T}^{-1}(\alpha) \geq F_S^{-1}(\alpha) + F_T^{-1}(\alpha), \quad (1)$$

where $F_S^{-1}(\alpha), F_T^{-1}(\alpha), F_{S \cup T}^{-1}(\alpha)$ are fractiles for probability α of distribution functions for coalitions S and T, and also for a coalition $S \cup T$, that is formed in the result of their alliance. A game will be named not strongly superadditivity (almost superadditivity) if condition (1) is fulfilled for all α, beginning from one α'.

In according with similar principles we can introduce a concept of convex (almost convex) SCG, understood under this games for which $\forall\alpha\ (\exists\alpha' : \forall\alpha \geq \alpha')$:

$$F^{-1}_{S\cup T}(\alpha) \geq F^{-1}_{S}(\alpha) + F^{-1}_{T}(\alpha) - F^{-1}_{S\cap T}(\alpha)\,. \tag{2}$$

Finally approaches to definitions of solutions can be directly carried over SCG. In particular a stochastic C_α -core should be read as set of imputations:

$$C_\alpha(\tilde v) = \{x \in R^{|I|} \mid \quad \forall S \neq \varnothing, I: \quad \begin{array}{l} \mathbf{P}\{\tilde v(S) \leq x(S)\} \geq \alpha; \\ \mathbf{P}\{\tilde v(I) \geq x(I)\} \geq \alpha\}. \end{array} \tag{3}$$

In our judgment approaches to model development of PPP's relationships, based on suggestion that a relationship between participants of PPP can be presented as stochastic cooperative game, seem to be enough useful and eventually, they allow to receive important conclusions in economical theoretical plan concerning regularities of development of such a form, found on interaction of government and private industry.

ВЕСТНИК САНКТ–ПЕТЕРБУРГСКОГО УНИВЕРСИТЕТА

Серия 10
ПРИКЛАДНАЯ МАТЕМАТИКА, ИНФОРМАТИКА, ПРОЦЕССЫ УПРАВЛЕНИЯ

Ответственный редактор серии:
Л.А. Петросян
Зам. ответственного редактора:
Д.А. Овсянников
С.В. Чистяков

Издательство Санкт-Петербургского университета

Virtual Implementation of Social Choice Function of Linear Aggregation

Nikolay A. Korgin

Institute of Control Sciences of Russian Academy of Sciences,
65 Profsoyuznaya Street
117997 Moscow GSP-4, Russia
Tel: +7-095-335-60-37; e-mail: nkorgin@ipu.ru.

Keywords: *Strategy-proof voting, Virtual implementation, Direct mechanisms*

Abstract: *Collective decision problem in which preferences of agents are private information is considered. In situation, when desired social choice function is not dominant strategy incentive one can try to virtually implement it – try to find another social choice function, that can be implemented in dominant strategies and arbitrary close to desired one. It is proved that iff social choice function is linear aggregation of agents best alternatives, then its best virtual implementation is direct mechanism, constructed from it.*

Collective decision problem in which preferences of agents are private information is considered. For society N with n agents facing a set A of alternatives, a social choice function determines what alternative to choose for each possible profile of preferences in case when there is no private information. Situation, when social choice function (SCF) is not dominant strategy incentive compatible (or strategy-proof) is considered – there is no mechanism that can implement it in dominant strategies [see Jackson (2001)]. And it can be very desirable SCF - for example, it can maximize total utility of the society (sum of utilities of agents) for any possible profile of preferences. In this case one can try to virtually implement [see, for example Jackson (2000)] this SCF – try to find another SCF, that can be implemented in dominant strategies and arbitrary close to desired one.

I consider the setting when the set of feasible alternatives A is a Cartesian product range in $\Re^m$ (M – set of dimensions) and agent's preferences multidimensional single-peaked (each agent $i \in N$ has unique most preferred alternative – agent's top $\tau_i = (\tau_i^1, \dots, \tau_i^m)$) with the added requirement that the unconstrained maximal element of these preferences belongs to A. Due to Barbera, Masso and Serizawa (1998) and Border and Jordan (1983), any SCF in this setting is strategy-proof if and only if it is a generalized median voter scheme. This result follows Moulin's (1980) initial analysis of the one-dimensional case ($A \subseteq \Re$). According to all this studies and Bossert and Weymark (2008) if SCF is strategy-proof, then it is tops-only – result of SCF depends only from profile $\tau = (\tau_1,\ldots,\tau_n)$ of agent's tops.

I consider class Π of tops-only SCF $f : A^n \to A$, that satisfies following conditions:

1. monotonic over agent's tops;
2. continuous over agent's tops;
3. satisfies unanimity condition;
4. separable over dimensions (for multidimensional case).

For any SCF f, satisfying this condition I characterize strategy-proof SCF $\tilde{f} : A^n \to A$, that is virtual implement it, minimizing following criteria:

$$K(f,\tilde{f}) = \max_{\tau \in A^n} \left\| f(\tau) - \tilde{f}(\tau) \right\|_{L_1}$$

Using this characterization and results for equivalent direct mechanisms in Burkov, Iskakov, Korgin (2008), I extend Burkov's (1989) initial results for SCF $f(\tau) = \dfrac{1}{n}\sum_{i=1}^{n} \tau_i$ in one-dimensional case to multidimensional case and class of SCF of linear aggregation:

$$f(\tau) = (f_1(\tau^1),\ldots,f_m(\tau^m)), m = \# M,$$

$$\forall k \in M\, f_k(\tau^k) = \sum_{i=1}^{n} \alpha_i^k \tau_i^k,$$

$$\forall k \in M \sum_{i=1}^{n} \alpha_i^k = 1, \forall i \in N\, \alpha_i^k \geq 1.$$

Following result is proved.

Theorem. SCF $f^*(\tau) = \arg\min_{\pi(\tau)\in\Pi} K(f(\tau),\pi(\tau))$ is Nash implemented by mechanism { A^n, f } if f is SCF of linear aggregation.

Other words – iff SCF is linear aggregation of agents best alternatives, then its best virtual implementation is direct mechanism, constructed from it.

Practical corollary of this theorem is that any SCF of linear aggregation may be considered as weakly manipulated - best virtual implementation of such SCF f - is direct mechanism, constructed from this SCF - { A^n, f }. If such SCF is used for collective decision problem in case of full information, then in case of private information it is no need to change rules of decision making.

References

[1]. Barbera S., Masso J., Serizawa S. Strategy-proof voting on compact ranges // Games and Behavior. 1998. Vol. 25.
[2]. Border K., Jordan J. Straightforward elections, unanimity and phantom voters // Review of Economic Studies. 1983. Vol. 50.
[3]. Bossert W., Weymark J.A. Social choice (new developments) / The New Palgrave Dictionary of Economics. Second Edition. Eds. Steven N. Durlauf and Lawrence E. Blume. Palgrave Macmillan, 2008.
[4]. Burkov V., Danev B., Enaleev A. Bol'shie sistemy: modelirovanie organizatsionnykh mekhanizmov, Nauka, Moscow 1989.
[5]. Burkov V., Iskakov M, Korgin N, Multicriterion competitiveness evaluation and the ranking of economic objects operating under various uncertainty conditions // Problemy upravlenia. 2008, Vol. 4.
[6]. Jackson M. "A Crash Course in Implementation Theory," Social Choice and Welfare, 2001 Vol. 18
[7]. Jackson M. "Mechanism Theory", The Encyclopedia of Life Support Systems , EOLSS Publishers Co. Ltd, United Kingdom, 2000.
[8]. Moulin H. On strategy-proofness and single-peakedness // Public Choice. 1980. Vol. 35.

МАТЕМАТИЧЕСКАЯ ТЕОРИЯ ИГР И ЕЕ ПРИЛОЖЕНИЯ

Ответственный редактор:
Л.А. Петросян
Зам. ответственного редактора:
В.В. Мазалов
Ответственный секретарь:
Н.А. Зенкевич

Выходит ежеквартально с января 2009 года

Differential Game of Advertising Management: Fashion Market

Anastasia Koroleva[1] and **Nikolay Zenkevich**[2]

[1]*Faculty of Applied Mathematics and Control Processes*
St. Petersburg University
Universitetskii prospekt 35, Petergof
St. Petersburg, 198504, Russia
anakorol@gmail.com

[2]*Graduate School of Management*
St. Petersburg University
Volkhovsky Per. 3
St. Petersburg, 199004, Russia
zenkevich@gsom.pu.ru

Keywords: *Differential game, Oligopoly dynamic competition, Profit maximization, Advertising strategy, Nash equilibrium*

Abstract: *The differential game of oligopolistic competition by advertising on fashion market is considered. The companies try to maximize their profit over an infinite horizon. The Nash equilibrium and the optimal expression for the optimal strategy have been found.*

This paper considers the model of competition by advertising. Fashion market is investigated. The model uses some of its specifications. The first specification is that customers are divided to those who are loyal to the whole fashion market and to those who are not loyal. The loyal customers to the market are those who buy more than two items a year in each firm from the industry. The second specification is that it is necessary to advertise the products of the firm only for the potential customers.

It is supposed that there are n firms (players) on the market. The model assumes that market has its maximum sales potential N. The firms use advertising $a_i(t),\ i = 1,...,n$, as a strategic instrument with an effectiveness β_i to find new customers and to increase their sales $s_i(t)$. Players want to maximize their discounted profit V_i over an infinite horizon:

$$\max_{a_i > 0} \int_0^{\infty} e^{-r_i t}(q_i s_i - a_i^2)\, dt,$$

where q_i is unit contribution and r_i is discount rate of firm i. The cost of advertising is intended to be quadratic.

The model considers the dynamic demand as

$$\dot{s}_i = \beta_i a_i \sqrt{N - s_i} + \rho \sum_{j=1}^{n} s_j, \quad i = 1,...,n,$$

where ρ is the rate of the loyal customers.

This model is a modification of the Vidale-Wolfe model, which takes in account the specifications of the fashion market.

Using the models for V_i and $\dot{s}_i$ we set the dynamic differential game:

$$\begin{cases} \dot{s}_i(t) = \beta_i a_i \sqrt{N - s_i} + \rho \sum_{j=1}^{n} s_j, \quad i = 1,...,n, \\ \max_{a_i > 0} \int_0^{\infty} e^{-r_i t} (q_i s_i - a_i^2)\, dt. \end{cases}$$

The Nash equilibrium and the expression for the optimal advertising strategies were found for this problem. Also the comparative analysis for the case of the symmetric competition has been performed.

References

[1]. Crane, D. Fashion in Science: Does It Exist? Social Problems. (1969), 16, 433-441.

[2]. Vidale, M. and H. Wolfe (1957). An operations research study of sales response to advertising. Operations Research, 5, 370-381.

[3]. Jorgensen S., and S. Zaccour (2004). Differential games in marketing. Kluwer Academic Publishers, Boston/ Dordrecht/ London.

[4]. Erickson, G. M. (2009). An oligopoly model of dynamic advertising competition. European Journal of Operational Research., 197, 374-388.

[5]. Prasad A., and Sethi S. (2003). Dynamic optimization of an oligopoly model of advertising. <http://ssrn.com/abstract=960117>.

[6]. Andreeva A. N. (2008). Disigner's brands on fashion market. Saint- Petersburg State University Publishing house. (In Russian.)

Market Equilibrium in Negotiations and Growth Models

Arkady Kryazhimskiy

International Institute for Applied Systems Analysis,
Laxenburg, Austria
and
Steklov Mathematical Institute,
Russian Academy of Sciences,
Moscow
kryazhim@iiasa.ac.at

The notion of the market equilibrium has been used in research on emission reduction games. An underlying phenomenon is the lack of well-established currency in the market of emission reductions; the market operation requires therefore special "rules of the game", acceptable for all market players. One of the possible "rules of the game" suggests bilateral agreements on the rates of emission exchange. A set of bilateral emission exchange rates is understood as the market equilibrium if each player's emission reduction exchanged to the emission reductions of all the other players maximizes the player's utility under that set of exchange rates. Finding the market equilibrium without revealing the individual utilities can constitute the goal of various multi-round negotiation processes.

In this talk, a brief outline of emission reduction games will be followed by a presentation of a dynamic game of interacting economic agents. The model suggests that each agent produces a unique good vital for the other agents. In that group of "monopolists" a market price formation mechanism can hardly emerge; the economic development is driven by bilateral rates of exchange in goods. The notion of the market equilibrium is used to define the growth dynamics acceptable for all the agents. The market equilibrium is formed as a set of time-varying bilateral exchange rates that allow each agent to produce his/her good via maximizing his/her individual integrated utility in response to the good offers from the other agents. In the talk, an illustrative example will be provided, and robustness properties of the market equilibrium will be discussed.

Tax Auditing Models With The Application of Theory of Search

Suriya Kumacheva

Saint-Petersburg University
Russia
s_kumach@mail.ru

Keywords: *Tax control, Tax evasions, Search games, Search theory*

Abstract: *One of the most important aspects of modeling of taxation – tax control is studied in the network of game theoretical attitude. Realization of systematic tax audits is considered as the interaction of two players – hiding (T – taxpayer) and seeking (A – authority) – in different statements of the search game. When the problem is stated it is assumed, that tax authorities know due to indirect signs, that some taxpayer T evaded from taxation in the given period (he put down his income too low in his tax declaration). Also it is assumed, that this taxpayer is quite a large company, having many branches. Company can distribute the hidden income among all or some of its branches or can hide it in the income of one of its branches. Different game cases, which conditions are determined by various possibilities of the evading taxpayer and the tax authority, are considered. In the conditions of the games considered, tax authorities A solve the problem of maximization of its income, using mixed strategies, which allow making optimal (in sense of evasion search) tax audits. Taxpayers T aspire to hide their income with the help of mixed strategies, allowing to reduce a probability of revealing of their evasion. While solving the task of tax audits it should be taken into account, that practically tax authorities are hierarchic structures and are solving tasks on different levels (federal, regional, district etc.). That's why in the considered game-theoretical examples the solution of this task is made with the use of the search theory of immobile chain with the help of a big search system. Tax authorities' task is to maximize the probability of revealing of evasion is solved with the help of the optimal (in the given sense) distribution of search efforts on the given period of time. A company solves the opposite task: to distribute evasion in the way, that minimizes probability of evasion revealing. Thus, different possibilities of solving the task of tax evasion search are considered..*

Optimal search of possible tax evasion and dependence of this search's results on spent resources (number of searching units, finance etc.) are important questions of modeling of the tax auditing ([1,2]). The task of the search of tax evasions with the help of the different configurations of the search games ([3]), where the players are tax authority (A - authority) and taxpayer (taxpayers) (T - taxpayers) is studied below.

At the end of the given tax period each taxpayer declares income and pays tax in compliance with them. Let's assume, that tax authority knows due to indirect signs, that some firm evaded from taxation in the given period (it put down lower than real income in the tax declaration). Let's also assume, that this firm is quite a large company, having many branches. The company can distribute the concealed income among all or some of its branches.

The tax authority has hierarchical structure and solves tasks on different levels (federal, regional, distirct e t.c.). Taking this feature into account, use of the described in [5] theory of search of immobile chain with the help of a big search system becomes acceptable to the task of tax audits.

Let's suppose, that the firm conceals income only in one of n branches. Let k be the number of this branch; the tax autority doesn't know it. The search of the object (concealed income) is realized in the discrete space of n branches. Speaking, that the tax authority is a big search system, as in [5], let's assume, that the activity of the system's separate unit can be defined with the help of function of density or strategy of search $\lambda_i(t)$. The vector $u = (u_1, \ldots, u_n)$ characterizes a distribution of a tax evasion among branches (each its component u_i is the probability of evasion of the i-th branch). A posteriori distribution of the evasion is characterized by the function

$$u(t) = (u_1(t), \ldots, u_n(t)) .$$

After some transformations the probability of revealing of the company's evasion in the time interval $(0;T)$, which is a period of limitation for tax crimes, is found:

$$P(T) = 1 - \sum_{i=1}^{n} u_i \exp(-\int_0^T \lambda_i(\tau) d\tau) . \tag{1}$$

Along with $\lambda_i(t)$ function $\phi_i(t)$ is considered, which is the number of search resources, spent on the revelation of the tax evasion in the i-th branch during the time interval $(0;T)$. Then (1) gets the form

$$P(T) = 1 - \sum_{i=1}^{n} u_i \exp(-\phi_i(T)) . \tag{2}$$

The tax authority's task is such a distribution of search resources $\phi_i(T)$ in the branch in the given period of time $(0;T)$, that maximizes the probability (2). The company's task is opposite: to distribute an evasion in such a way, that minimizes probability of revealing of evasion $P(T)$.

Then several models of the search of evasion, which based on the matrix search games, described in ([3]), are considered:

• The model of search of evasion, which is concentrated in the one branch;

• The model of search of evasion, which is concentrated in one or two branches;

• The model with the possibility of auditing of the nearest branches only;

• The model with the distribution of the concealed income among m branches;

• The model with the distribution of equal or different parts of the concealed income among m branches.

Within the bounds of the mentioned models the search of the players' optimal mixed strategies and respective profits is realized. To solve the dependence of the evasion revealing probability on the spent search resources is taken into account in the solution of the appropriate games.

The considered games demonstrate different possibilities of the application of the search theory to the modeling of the tax control.

References

[1] Boure V., Kumacheva S. A model of audit with use of statistical information about taxpayers' income. Vestnik Sankt-Peterburgskogo universiteta. Seria 10. Vol. 1-2, june 2005. Izdatelstvo SPbGU. Saint-Petersburg. 2005.

[2] Vasin A, Morozov V. The Game Theory and Models of Mathematical Economics. MAKSpress. Moscow. 2005.

[3] Petrosyan L., Garnaev A. The search games. SPb: Izdatelstvo SPbGU, 1992.

[4] Petrosyan L., Zenkevich N., Semina E. The game theory. Vysshaya shkola, Knizhnyj dom "Universitet". Moscow. 1998.

[5] Hellman O. Introduction to the optimal search theory. Optimization and operations research. Moscow: Nauka, 1985.

Automatization of Processing Input Data in Computational Programs

Sergey S. Kumkov[1] and **Denis Yu. Sannikov**[2]

[1]*Institute of Mathematics and Mechanics, Ural Branch,*
Russian Academy of Sciences,
Ekaterinburg, Russia,
sskumk@gmail.com

[2]*Ural State University,*
Ekaterinburg, Russia,
denis.v.kepke@gmail.com

Keywords: *Numerical methods, Computational programs, Input data, C#, Linear differential games*

Any computational program needs to be fed with input data. And this does not depend on the language and/or environment, which with the program is written. The simplest and the most wide-spread format of input data is ‘‘plain text’’, when the input file is just a serie of numbers, which sense is defined by their positions in the serie. For example, if our program solves a linear differential equation $\dot{x} = Ax$ with an initial condition $x(0) = y$, it needs $n^2 + n + 1$ numbers: at first, an integer n describing dimension of the vector x, then n^2 reals describing the matrix A, and, finally, n reals describing the initial value y.

Such a simplicity of the format makes reading the file very easy: at first, read one number, then n^2 numbers, and finally n numbers more. And since there is no any additional marking information, all correctness checking must be made when preparing the input data. But when the amount of the data is large, the user can easily err; in our example, if there are only $n^2 + n$ numbers in the file, it is very difficult to understand what exactly number is lost.

Another problem is that if we wish to solve a non-autonomous equation $\dot{x} = A(t)x$, then our file should include both numbers and formulae, and the programmer (who is not a professional usually, but a mathematician) must write a formula parser and evaluator.

The main objective of our work is to create a library, which undertakes reading the input file, parsing descriptions of data, storing read objects, and providing convenient access to the stored values (including recomputing formulae according to the current state of variables defining these formulae). Such a library is developed using C#. This language has been chosen due to the following. At first, it provides a very flexible and comprehensive realization of object-oriented paradigm allowing to create ‘‘smart’’ object, which work almost in the same way as usual variables and hide all internal processes from the programmer. At second, it provides portability of programs, because the .Net platform is fulfilled both for Windows and Linux popular operational systems.

The description language for the input file is based on XML, because the latter is very convenient both for preparing data and for reading them. Description has two parts. The first one, a file of the structure of the input data, tells what data are necessary for a program. In out example with differential equation, it includes description of three variables: an integer n, a real matrix A of size $n \times n$, and a real vector y of size n. (So, one can see, that values of some variables can define parameters of others.) The second part contains certain values of the variables described in the structure file; the values corresponds to a certain example to be computed (in the case of equation, the file of data contains numerical of formula data defining a concrete differential equation). Of course, the file of structure is unique for each program, and files of data can be multiple.

The library worked out is used in a serie of programs for solving linear differential games. The talk gives some numerical examples computed by these programs.

Plans for future extending this work includes developing a graphical environment for convenient preparation of input data for programs, which use our library.

Quality-Price Model with Vertical Differentiation: the Effect of Cooperation

Denis V. Kuzyutin[1] and **Elina Zhukova**[2]

[1]*International Banking Institute (Saint-Petersburg),*
Saint-Petersburg University
Russia
Kuzyutin@ibispb.ru

[2]*Saint-Petersburg University*
Russia
Zhukova_elina@mail.ru

Keywords: *Industrial organization, Product differentiation, Cooperation, Punishment strategies, Subgame perfect equilibrium.*

Abstract: *The paper describes a quality-price competition duopoly model. The firms interact in a two-staged game, the cases of non-cooperation and cooperation are considered. The existence and stability of the equilibrium in punishment strategies is analyzed.*

The paper examines equilibrium solutions for quality-price competition models of product differentiation. The aim is to compare different cases of strategic interactions between the players.

We consider a duopolistic market of two firms producing vertically differentiated product. The firms compete in qualities and prices. The competition between firms takes place in a two-staged game. In the 1st stage they decide on the quality $q_i,\ i=1,2$ to produce (let $q_1 < q_2$). At this stage each firm faces R&D costs. In the 2nd stage choose simultaneously their prices $p_i,\ i=1,2$. Before the 2nd stage begins, $q_i,\ i=1,2$ become a common knowledge. Each firm's strategy is maximizing its profit. There is a continuum of consumers characterized by a parameter t - willingness to pay, uniformly distributed within the interval: $t \in [0,1]$. [Motta, 1993; Ronnen 1991]

The model is extended to the case of cooperation of the players. Before the 1st stage of the game begins, players communicate with each other and decide to form some kind of cartel. Two firms form a coalition when the main goal of each player is maximizing not its own profit but summarized profit of both firms. Then some profit allocation procedure takes place. This “cooperative” solution $(q_1^C, q_2^C, p_1^C, p_2^C)$ is compared with the non-cooperative one: (q_1, q_2, p_1, p_2).

Cooperation brings better payoffs for both players only if each firm plays honestly and chooses strategies $(q_i^C, p_i^C),\ i = 1,2$ that bring optimal solution. In order to stabilize the collusion the concept of punishment strategies is used. Thus if both players play honestly they choose cooperative qualities at 1st stage of the game: $(q_i^C, p_i^C),\ i = 1,2$. Let the 1st player to abort the agreement and deviate from the “cooperative” solution: $q_1 \neq q_1^C$. In this case at the next stage of the game the 2nd firm plays against its rival and chooses $p_2 = p_2^P$ that minimizes the profit of the 1st firm: $\Pi_1 \to \min$.

The paper considers several variants of strategic interaction between the firms, comprehension takes place. The question of Pareto optimality of the solutions is examined.

References

[1]. L. Petrosjan, D. Kuzyutin. Consistent solutions of positional games. St-Petersburg University Press, 2008, 335p. (in Russian).

[2]. Motta M. Endogenous quality choice: price vs. quantity competition // The Journal of Industrial Economics. 1993. Vol. XLI, No.2.p.113-130.

[3]. Ronnen U. Minimum quality standards, fixed costs and competition // The Rand Journal of Economics. 1991.Vol.22,No.4.p.490-504.

[4]. Shaked A., Sutton J. Relaxing Price Competition throught Product Differentiation. Review of Economic Studies, 1982, Vol.49,p.3-14.

[5]. Tirole J. The theory of industrial organization. The MIT Press, 1997.

Testing and Statistical Games

Mikhail Lutsenko

Mathematics and simulation,
St. Petersburg Transport University,
ML4116@mail.ru

Keywords: *Test reliability, Antagonistic game, Statistical game, Randomize decision function, The worst priori distribution*

Abstract: *In the paper some game models of testing are constructed and their reliability is shown. The matters of investigation are statistical games between Nature and Statistician. Solution of the games (the worst a priori distribution, optimal decision function) is found. There is no assumption about distribution of level of knowledge of test-takers. The method of game solution is simple in realization. The papers of author [1, 2] are used.*

Introduction

The main goal of any testing is assessment intended to measure the test-takers' knowledge, skills, aptitudes, or classification in many other topics. The problem is especially important if test score is used for administrative decision. There are many papers devoted to this theme for example [3]. But estimation of accuracy and reliability is made with additional assumption that level of knowledge of examinees has normal distribution. However it is difficult to agree with this preposition. At the test control of level of knowledge of pupils the essential part of training is devoted to preparation for testing. But, knowing the form of the test pupils can so transform their knowledge that the objective measure of knowledge is complicated. Thus, we have a conflict situation (game), which participants are the Statistician and the Test-taker. The second wants to get the highest test score spending minimal time for training. The first wants to find an accuracy estimation of level of knowledge.

Solution of finite statistical games

Let's reduce a classification problem to a problem of estimation. For this purpose we introduce following notations. We denote a finite set of possible levels of knowledge of the pupil by $\Theta = \{\theta_1, \theta_2, \ldots, \theta_m\}$ (set of parameters); set of possible values of a random variable $\mathbf{X}_\theta$ by $X = \left\{x_1, x_2, \ldots, x_N\right\}$ (set of observation) and family of distributions of the variable $\mathbf{X}_\theta$ on set X by $\left\{P_\theta(x)\right\}_{\theta\in\Theta}$. So $P_\theta(x)$ returns probability that test score of a pupil $\mathbf{X}_\theta$ is equal to x if his level of knowledge is θ. We denote set of admissible estimations of knowledge of the pupil by $D = \left\{d_1, d_2, \ldots, d_n\right\}$ (set of decisions). Function $\delta : X \to D$ is called decision function. Then $\delta(x)$ is the decision of the Statistician if value of a random variable $\mathbf{X}_\theta$ is equal to x. We denote set of decision functions by $\mathbf{D} = D^X$. It is possible to present each decision function in the form of a vector $\delta = \left(\delta_1, \delta_2, \ldots, \delta_N\right)$, where $\delta_k = \delta(x_k) \in D$.

With each decision $d \in D$ we tie a subset $\Theta(d) \subseteq \Theta$ of levels of the knowledge admissible for this decision. By the set family of subsets $\left\{\Theta(d)\right\}_{d\in D}$ we construct a pay-off function of the Statistician:

$$h(d,\theta) = \mathbf{1}_{\Theta(d)}(\theta) = \begin{cases} 1, if \quad \theta \in \Theta(d), \\ 0, if \quad \theta \notin \Theta(d). \end{cases}$$

Thus, the pay-off of the Statistician is equal to unit only if the examinee is correctly estimated.

Let's make statistical game $\Gamma = \left\langle \mathbf{D}, \Theta, H \right\rangle$, where pay-off function has the following form

$$\mathbf{P}\Big(\theta \in \Theta\big(\delta(\mathbf{X}_\theta)\big)\Big) = \sum_{x\in X} P_\theta(x) \cdot h(\delta(x), \theta) = H(\delta, \theta) .$$

Value of the game is probability that measurement of examinee is correct for any a priori distribution of parameter θ.

It is well-known that Statistician can use mixed strategies of the form $\mu = \left(\mu_1, \mu_2, \ldots, \mu_N\right)$, where $N = |X|$ is a number of elements of set X and μ_k, $k = \overline{1,N}$ are probability measures on the set of decision D.

If the sets X, D, Θ are finite and numbers of their elements equal respectively to N, n, m, then Γ is matrix game with pay-off matrix of the size $Nn \times m$. We denote elements of matrix B by $b_{i,j} = h(\theta_i, d_j)$, $i = \overline{1,m}$; $j = \overline{1,n}$, nonzero elements of diagonal matrix Λ^k by $\lambda^k_{i,i} = P_{\theta_i}(x_k)$, $i = \overline{1,m}$, $k = \overline{1,N}$; elements of randomized decision function by $\mu = \left(\mu_1, \mu_2, \ldots, \mu_N\right)$; $k = \overline{1,N}$ with $\mu_k = \left(\mu_k^1, \mu_k^2, \ldots, \mu_k^n\right)^t$; vector a priory distribution of parameter θ by $\nu = \left(\nu_1, \nu_2, \ldots, \nu_m\right)^t$; column of *m* units by $\mathbf{1}_m$.

To solve matrix game we construct a pair of dual problems. We find the best randomized decision function $\mu = \left(\mu_1, \mu_2, \ldots, \mu_N\right)$ and the worst a priori distribution ν from the first and the second problem respectively. The common value of two problems is the value of game Γ.

Primal problem	Dual problem
$v \to \max,$ $\sum_{k=1}^{N} \Lambda^k B \mu_k \geq v \mathbf{1}_m$ $\sum_{j=1}^{n} \mu_k^j = 1$; $\mu_k^j \geq 0$ $k = \overline{1,N}$; $j = \overline{1,n}$;	$v = \sum_{k=1}^{N} u_k \to \min,$ $\nu^t \Lambda^k B \leq u_k \mathbf{1}_n^t$; $k = \overline{1,N}$; $\sum_{i=1}^{m} v_i = 1$.

A lot of methods of solution of problems of linear programming are known. The dynamic method that is worked out by the author [2] especially for statistical games with threshold pay-off functions would be the most convenient here. But in simple cases the statistical game can be solved by means of MS Excel. Though this method often does not give the exact solution, but it always specifies feasible solutions of problems and the upper and lower estimations of matrix game's value.

Example. We suppose that a test contains 10 items and Statistician makes a decision by results of the test. Observation set X has 11 numbers from zero to ten. The probability of correct response to an item of test is a measure of respondents' knowledge. The possible values of levels of knowledge form a parameter set $\Theta = \left\{ 0,95;\ 0,85;\ 0,75;\ 0,65;\ 0,55;\ 0,45;\ 0,35;\ 0,25;\ 0,15;\ 0,05; \right\}$. The probability that examinee gives exactly *k* corect responses is given by formula

$$P\left(\mathbf{X}_\theta = x\right) = \binom{10}{x} \cdot \theta^x (1-\theta)^{10-x},\ x = \overline{0;10}$$

Thus each examinee has one of 10 possible levels of knowledge of value which vary from 95 % to 5 %. In this example we will consider that decision set and parameter set coincide ($\Theta = D$). The admissible interval includes only those values of parameter θ which deviates from d on distance not more than 10 %, that is

$$h(d,\theta) = \mathbf{1}_{\Theta(d)}(\theta) = \begin{cases} 1,\ if\ \mid i - j \mid \leq 0,1; \\ 0,\ if\ \ \mid i - j \mid > 0,1. \end{cases}$$

Let's make statistical game $\Gamma = \langle \mathbf{D}, \Theta, H \rangle$ and solve it in the mixed strategies by means of MS Excel. The pay-off matrix in the game has the size 110×10. We get upper and lower estimations of value of game (0,771 and 0,788), randomized decision function and the worst aprioristic distribution of parameter θ.

The received values of game can be improved if a priori distribution of parameter is known. Let's notice that value of game change at increase in number of items in the test a little.

References

[1] Lutsenko M. M., Maloshevskii, S.G., (2003), Minimax confidence intervals for the binomial parameter. Jornal of Statistical Planning and Inference 113, 67-77.

[2] Lutsenko M. M., Ivanov M. A., (2000)), Minimax confidence intervals for the parameter hypergeometric distribution. Automat. Remote control 61 (7) part 1, 1125-1132. (translated from Avtomatika i Telemekhanika (7), 68-76).

[3] Gronlund, N. (1993) "How to make achievement tests and assessments," 5th edition, NY: Allyn and Bacon

The Generalized Kalai-Smorodinsky Solution for the Multicriteria Problems

Andrey N. Lyapunov

St.Petersburg Institute for Economics and Mathematics
Russian Academy of Science,
Tchaikovsky st.1, 191187 St.Petersburg, Russia
anlyapunov@yandex.ru
(812)5426152

Keywords: *Kalai-Smorodinsky solution, Multicriteria problems, Minkovsky function*

Abstract: *The Kalai-Smorodinsky solution for the bargaining problem is generalized to the multicriteria problems.*

Introduction: An extremal property of the KS-solution.

The KS-solution has been proposed in [1]. See too [2], [3].

Let $X \subset \mathbb{R}^n_+$ be a convex compact and comprehensive set. Put

$$a_i(X) = \max\{x_i \mid x \in X\}, \quad a(X) = (a_1(X),\dots,a_n(X)),$$

and

$$\lambda = \max\{t \mid ta(X) \in X\}.$$

Then the KS-solution $x^* = KS(X)$ is determined by the following manner:

$$x^* = \lambda a(X).$$

Let $\phi : \mathbb{R}^n_+ \to \mathbb{R}_+$ be the Minkovsky function of the set X .

Lemma 1. KS-solution x^* is the solution of the following optimization problem:

$$\phi(a(X) - x) \to \min, \quad x \in X . \tag{1}$$

The vector $a(X) - x$ is called *the loss of the vector* x .

The multicriteria problems with nontrasferable utilities (NTU-problems).

Definition. *The NTU-problem* is a pair (X, F) , where $X \subset \mathbb{R}^m$ is the set and $F : X \to 2^{\mathbb{R}^n}$ is the multi-valued map.

Further we shall suppose that the map F fulfils to following conditions:

C1. The graph G of the map F is the convex compact set in $\mathbb{R}^n \times \mathbb{R}^m$.

C2. $F(x) \subset \mathbb{R}_+^m$ is the comprehensive set for all $x \in X$.

Put

$$Y = \bigcup_{x \in X} F(x).$$

The set $Y \subset \mathbb{R}^m$ is convex compact comprehensive set. Let ϕ_x be the Minkovsky function of the set $F(x)$ and ϕ be the Minkovsky function of the set Y.

Lemma 2. $\phi(y) = \min_{x \in X} \phi_x(y)$.

Definition. *The KS-solution of the NTU-problem* (X, F) is the pair $(x^*, y^*) \in G$ such that y^* is the KS-solution for Y.

Let (X, f) be a TU-problem.

Definition. The NTU-problem (X, F_f) where $F_f(x) = \{y \in \mathbb{R}_+^n \mid y \le f(x)\}$ is called *the NTU-problem corresponding to TU-problem* (X, f).

The concave TU-problem

Consider the TU-problem (X, f), where $X = \mathbb{R}^n$ and $f : \mathbb{R}^n \to \mathbb{R}^m$ are the strictly concave functions having the maxima. The class such problems (or functions) we denote by C_n^m. Put

$$x^i = \arg\max_x f_i(x), \quad a(f_i) = f_i(x^i), \quad i = 1, \ldots, m, \quad a(f) = (a(f_1), \ldots, a(f_m)),$$

$$q(f_i) = \min_k f_i(x^k), \quad i = 1, \ldots, m, \quad q(f) = (q(f_1), \ldots, q(f_m)).$$

$$Z = \{z \in \mathbb{R}^m \mid q(f) \le z \le a(f)\}, \quad X = f^{-1}(Z).$$

Let $\mathbb{R}(f)$ be the set of the Pareto-optimal points in the problem f.

Lemma 3. *The set* X *is the convex compact set and* $\mathcal{E}(f) \subset X$.

Let (X, F_f) be the NTU-problem for the TU-problem f and ϕ the Minkovsky function for (X, F_f).

Suppose that $q(f) = 0$.

Definition. *The generalized KS-solution* x^* *of the TU-problem* f is solution of the following optimization problem:

$$\phi(a(f) - f(x)) \to \min, \quad x \in X \tag{2}$$

(see (1)).

From definitions of the Minkovsky function and the NTU-problem (X, F_f) it follows that x^* fulfils to the following problem:

$$a(f) - f(x^1) \le \lambda(f(x^2) - q(f)), \quad \lambda \to \min, \quad x^1, x^2 \in X\,, \tag{3}$$

where $x^* = x^{1*}$.

Theorem. *For existing the KS-solution for the problem f it is necessary and sufficient that in problem (3) it is fulfilled $x^{1*} = x^{2*}$.*

References

[1] Kalai E., Smorodinsky M. 1975. Other solutions to Nash's Bargaining Problem.//Econometrica, 43, 513-518.

[2] Pechersky S.L., Yanovskaya H.B. 2004. Cooperative games: solutions and axioms. S.Petersburg, 459 p.(In Russian).

[3] Thomson W. Cooperative models of bargaining. 1994. //Handbook of game theory with economic applications. v.II, eds. Aumann R.J., Hart S., 1237-1284.

Game Theoretical Model of Exhibition Business

Svetlana Mamkina

Nexus Holding,
Russia
s.mamkina@gmail.com

Keywords: *Exhibition business, PMS vector*

Dynamic game theoretical model of competition in exhibition business is considered. The new approach based on PMS vector and coalitional participation of players is proposed. The theoretical results are illustrated by examples based on concrete data obtained from exhibition business. The main players in the game are different industrial associations, exhibition organizers and chambers of commerce from different countries in different sectors of economy.

Cooperative Solutions for a Group Pursuit Game between a Pursuer and m Evaders

Irina Marchenko[1], **Yaroslavna Pankratova**[2] and **Svetlana Tarashnina**[3]

[1]*International Banking Institute*
Russia
march-irina@yandex.ru

[2]*St.Petersburg University,*
International Banking Institute
Russia
yasyap@gmail.com

[3]*St.Petersburg University*
Russia
tarashnina@gmail.com

Keywords: *Group pursuit game, Nash equilibrium, Realizability area, Cooperative game, Core, Extended core*

In this paper a game of group pursuit in which players move on a plane with bounded velocities is studied. The game is supposed to be a nonzero-sum simple pursuit game between a pursuer and m evaders acting independently of each other. Here we assume that the evaders are discriminated and dictated the extremely disadvantageous behaviour by the pursuer who has an element of punishment at his disposal. The case of complete information is considered. This means that each player choosing control variables at each time instant t > 0 knows the moment t and his own and all other players' positions. This game was considered in [1] [2].

In paper [3] Nash equilibrium was constructed for this game and a maximum number of evaders that have to obey the pursuer was estimated. A notion of the punishment strategy of the pursuer was introduced in the paper. Furthermore, for every evader a realizability area of the punishment strategy was constructed.

In this work we propose to consider a cooperative version of the described game and define its solutions. It is important to get a reasonable answer to the following question: if cooperation is profitable for the players in this differential pursuit game. We consider all possible cooperation between the players in the game. In our interpretation, the pursuer can promise some amount of the total payoff to the evaders for cooperation with him. Our conjecture is that for the evaders this offer might be advantageous.

With every a nonzero-sum pursuit game we associate a corresponding game in characteristic function form. In fact we have the whole family of games depending on initial positions of the players. We consider the core [4] as a main solution concept in this game. In case the core of the game is empty the extended core [5] is considered as a solution concept. Note that the extended core is never empty. Moreover, we describe some conditions under which in this game there exists the nonempty core for any initial positions of the players. It is notable that there is an interconnection between existence conditions of the Nash equilibrium and non-emptiness of the core. Finally, we give several examples that illustrate the obtained results. On the top of this, the described approach is applied to the travel agency problem.

References

[1] Petrosjan L.A. Shirjaev V.D. Simultaneous pursuit of several evaders by one pursuer. Vestnik Leningrad Univ. Math. Vol 13. 1981.
[2] Tarashnina S. Nash equilibria in a differential pursuit game with one pursuer and m evaders. Game Theory and Applications. N.Y. Nova Science Publ. 1998. Vol. III, P. 115-123.
[3] Pankratova Ya., Tarashnina S. How many people can be controlled in a group pursuit game. Theory and Decision. Kluwer Academic Publishers. 2004. 56, P. 165-181.
[4] Scarf H.E. The core of an n-person game. Econometrica. 1967. 35, P. 50-69.
[5] Gomez, J.C. An Extension of the Core Solution Concept, working paper 04-01. University of Copenhagen. 2004. Available at: http://www.econ.ku.dk /wpa/pink/abstract/0401.pdf

General Time-Inconsistent Preferences and Cooperative Differential Games

Jesus Marin-Solano[1] and **Ekaterina V. Shevkoplyas**[2]

[1]*Universitat de Barcelona*
Spain
jmarin@ub.edu

[2]*St.Petersburg University*
Russia
katya_shev@mail.ru

Keywords: *Time-inconsistent preferences, Dynamic programming equation, Cooperative differential games*

Abstract: *Previous results on non-constant discounting in continuous time are extended to the the case of more general time-inconsistent preferences. A dynamic programming equation providing a time-consistent solution to the problem is derived. Cooperative differential games are studied within this generalized setting.*

Variable rate of time preferences have received considerable attention in recent years. The most relevant effect of non-constant discounting is that preferences change with time: an agent making a decision at time t has different preferences compared with those at time s, for $s \neq t$. Therefore, we can consider her/him at different times as different agents. An agent making a decision at time t is usually called the t-agent. A t-agent can act in two different ways: naive and sophisticated. Naive agents take decisions without taking into account that their preferences will change in the near future. Then, they will be continuously modifying their calculated choices for the future, and their decisions will be in general time inconsistent. The solution is obtained by solving the family of associated optimal control problems for each t-agent, and patching together the optimal decision rules at time t. In order to obtain a time consistent strategy, the t-agent should be sophisticated, in the sense of taking into account the preferences of all the s-agents, for $s > t$. Therefore, the solution to the problem of the agent with non-constant discounting should be constructed by looking for the subgame perfect equilibria of the associated perfect information sequential game with a continuous number of players (the t-agents) making their decisions sequentially. This prompts the use of a dynamic programming approach, applying the Bellman optimality principle. As a result, the standard Hamilton-Jacobi-Bellman in optimal control theory is replaced by the much more complicated dynamic programing equation (DPE) introduced in Karp (2007) (see also Marn-Solano and Navas (2009) and Ekeland and Lazrak (2008)).

In this paper we extend previous results in non-constant discounting in continuous time to the case of a general discount function $d(s,t)$, where t denotes the time in which the agent is taking her/his decision, so that $d(s,t)$ denotes how the t-agent discounts utilities obtained at time s, for $s > t$. If $d(s,t)$ is a function of the distance between s and t, $d(s,t) = d(s-t)$, then we recover the problem with non-constant discounting. For the general case we derive a DPE in a rigorous way whose solution describes the equilibrium rule. Our approach (in the spirit of and Marn-Solano and Navas (2010)) is similar to that in Ekeland and Pirvu (2008) for the non-standard stochastic optimal control problem with non-constant discounting. Our DPE generalizes the DPE obtained in Rincón-Zapatero (2009) for the simple variational problem in finite horizon in which the Lagrangian depends just on the velocities, $L = L(\dot{x})$. Although the notion of equilibrium is not so natural as in the approach introduced by Karp (2007), we reproduce the results for the case of non-constant discounting in a mathematical rigorous way, with the advantage that it allows for more general versions of the problem.

Finally, we address our attention to cooperative differential games. The notions of dynamically stable cooperation, time consistency and time consistent payoff distribution procedures (see e.g. Petrosyan and Zaccour (2003) or Yeung and Petrosyan (2006)) are analyzed for the general problem.

References

[1] Ekeland, I. and Lazrak, A. (2008). Equilibrium policies when preferences are time inconsistent.

[2] Ekeland, I. and Pirvu, T.A. (2008). On a Non-Standard Stochastic Control Problem.

[3] Karp, L. (2007). Non-constant discounting in continuous time. Journal of Economic Theory 132, 557-568.

[4] Marn-Solano, J. and Navas, J. (2009). Non-constant discounting in finite horizon: The free terminal time case. Journal of Economic Dynamics and Control 33 (3), 666-675.
[5] Marn-Solano, J. and Navas, J. (2010). Consumption and portfolio rules for time-inconsistent investors. European Journal of Operational Research 201 (3), 860-872.
[6] Petrosjan, L.A. and Zaccour, G. (2003). Time-consistent Shapley Value Allocation of Pollution Cost Reduction. Journal of Economic Dynamics and Control 27, 381 – 398.
[71] Rincón-Zapatero, J.P. (2009). Hopf-Lax formula for variational problems with non-constant discount. Journal of Geometric Mechanics 1, 357-367.
[8] Yeung, D.W.K. and Petrosyan, L.A. (2006). Cooperative Stochastic Differential Games. Springer.

Bargaining Powers, a Surface of Weights, and Implementation of the Nash Bargaining Solution

Vladimir D. Matveenko

St. Petersburg Institute for Economics and Mathematics RAS
Matveenko@emi.nw.ru

Keywords: *Bargaining powers, Nash bargaining solution, Implementation*

Abstract: *In the present paper new approaches to the problem of implementation of the Nash bargaining solution (N.b.s.) are proposed. Shapley (1969) introduced weights of utilities and linked the N.b.s. with utilitarian and egalitarian solutions. This equivalence leaves open a positive question of a possible mechanism of the weights formation. Can the system of weights be received as a result of a recurrent procedure of reconciliation of utilitarian and egalitarian interests? Can a set of feasible weights be a result of a game independent on a concrete bargaining situation? We answer these questions in the paper.*

The Nash bargaining problem takes an important place both in the theory of cooperative games and in practical applications of the game theory. In the present paper new approaches to the problem of implementation of the Nash bargaining solution (N.b.s.) are proposed.

We consider an *n* person bargaining problem with the feasible set $S \subset R_+^n$ and the disagreement point $d = 0$. Nash (1950) formulated a list of properties ("axioms') a symmetric N.b.s has to satisfy. A weaker set of axioms corresponds to a family of asymmetric N.b.s.'s with bargaining powers, $\bar{x} = \arg\max_{x \in S} x_1{}^{a_1} x_2{}^{a_2} \ldots x_n{}^{a_n}$ (here $a_1, \ldots, a_n$ are bargaining powers of the players). Nash (1953) initiated a, so called, Nash program or implementation, trying to construct new games (usually, non-cooperative) leading to the N.b.s. Shapley (1969) linked the N.b.s. with utilitarian and egalitarian solutions (see also Yaari, 1981 and Binmore, 1998, 2005 who discuss the N.b.s and the Shapley's approach in a broad context of social justice and social contracts). With bargaining powers, the benchmark result can be formulated in the following way.

Let the set S be restricted by coordinate planes and by a surface $\varphi(x_1, \ldots, x_n) = 0$, where φ is a strictly convex function, and let $a_1, \ldots, a_n$ be positive bargaining powers. Then the following two statements are equivalent.

1. *For a point* $\bar{x}$ *such weights* $\lambda_1,...,\lambda_n$ *exist that* $\bar{x}$ *is simultaneously (a) a "utilitarian" solution* $\arg\max_{x\in S}(\mu_1 x_1 + ... + \mu_n x_n)$ *with weights* $\mu_i = a_i\lambda_i$ *(* i = 1,…,n*), and*

(b) an "egalitarian" solution $\arg\max_{x\in S}\min_i \lambda_i x_i$ *or, what is the same, a point where*

$$\lambda_1\bar{x}_1 = ... = \lambda_n\bar{x}_n .$$

2. $\bar{x}$ *is an asymmetric N.b.s. with bargaining powers* $a_1,...,a_n$.

This equivalence leaves open a positive question of a possible mechanism of the weights formation. Can the system of weights be received as a result of a recurrent procedure of reconciliation of utilitarian and egalitarian interests? We construct such iterative procedure, which has a natural economic sense and leads to the set of weights corresponding to the N.b.s., if some natural sufficient conditions are satisfied; the latter are formulated in terms of the elasticity of substitution of function $\varphi(x_1,...,x_n)$ in point $\bar{x}$.

Another question is: can a set of feasible weights be a result of a game independent on a concrete bargaining situation? In reality, bargains often have a routine character and are accomplished in presence of an arbiter (such as a court or an international community). Decisions of the arbiter depend in much on assessments which relate to reputations of the players but are defined concretely in each concrete bargain

We consider a two-stage game. On the first stage the players form a surface Λ of admissible sets of weights. The weights are interpreted as admissible reputation assessments created by players by use of mechanisms of propaganda. On the second stage, an arbiter (the society) chooses an admissible set of weights $\lambda \in S$ and an outcome $x \in S$ to achieve

$$\max_{x\in S}\max_{\lambda\in L}\min\{\lambda_i x_i\} .$$

On the first stage each player is evidently interested in decreasing her weight and in increasing the opponent's weight. However, when forming the surface Λ, player *i* agrees to a decrease in the opponent's weight, λ_j, in a part of the curve at the expense of an increase in his own weight, λ_i, as far as player *j* similarly temporizes in another part of curve Λ. With constant bargaining powers, the process is modeled by an equal-elasticity-condition leading to the differential equation: $\frac{d\lambda_i}{d\lambda_j}\frac{\lambda_j}{\lambda_i} = -\frac{a_K}{a_L} = const$. Solving this equation we get the surface Λ described by equation

$$\lambda_1^{a_1}\lambda_2^{a_2}...\lambda_n^{a_n} = C = const .$$

On the second stage of the game, the set Λ leads automatically to the asymmetric N.b.s. with bargaining powers $a_1,...,a_n$.

References

[1] Binmore K. Egalitarianism versus utilitarianism. Utilitas, 10, 1998, 353-367.
[2] Binmore K. Natural justice. Oxford, Oxford University Press, 2005.
[3] Nash J.F. The bargaining problem. Econometrica, 18, 1950, 155-162.
[4] Nash J.F. Two-person cooperative games. Econometrica, 21, 1953, 128-140.
[5] Shapley L.S. Utility comparison and the theory of games. In: La decision: Agregation et dynamique des orders de preference. Paris, 1969,251-263.
[6] Yaari M.E. Rawls, Edgeworth, Shapley, Nash: Theories of distributive justice re-examined. J. of Economic Theory, 24, 1981, 1-39.

РОССИЙСКИЙ ЖУРНАЛ МЕНЕДЖМЕНТА

Главные редакторы:
В.С. Катькало
Д.Дж. Тисс

Издательство Высшей школы менеджмента Санкт–Петербургского университета

Bargaining Models with Correlated Arbitrators

Vladimir V. Mazalov[1] and **Julia S. Tokareva**[2]

[1]*Institute of Applied Mathematical Research*
Karelian Research Center of Russian Academy of Sciences
Pushkinskaya str. 11, Petrozavodsk 185910, Russia
e-mail: vmazalov@krc.karelia.ru

[2]*Zabaikalsky State Humanitarian Pedagogical University*
named after N.Tchernishevsky
Babushkin str. 129, Chita 672007,Russia
e-mail: jtokareva2@mail.ru

Keywords: *Final offer arbitration, Equilibrium, Optimal strategies, Correlation*

We consider a version of arbitration procedure with correlated arbitration. There are the players L and M who present their offers x and y ($y \leq x$) to some arbitration committee which consists of some members. Each member after observing the offers x and y decides which offer must be selected. Let the solutions of the arbitrators are presented by correlated random variables $\alpha_1,...,\alpha_n$ with continuous distribution function $F(a_1,...,a_n)$. Each arbitrator chooses an offer using the final offer arbitration scheme. After that the committee determines the final solution using the majority rule. The objective of the paper is to find the equilibrium in this arbitartion game and estimate the effect of correlation for the final solution.

For example, consider a case with two arbitrators and normal distribution

$$f(a_1, a_2) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)}(a_1^2 - 2\rho a_1 a_2 + a_2^2)\right)$$

where ρ - correlation factor between random variables a_1 and a_2 **.** Then the equilibrium is

$$x^* = -y^* = \frac{1}{4} + \frac{1}{2\pi} arctg\left(\frac{\rho}{1-\rho^2}\right).$$

References

[1]. Farber H. An analysis of final-offer arbitration. Journal of conflict resolution. 35, 683 (1980).

[2]. Chatterjee K. Comparison of arbitration procedures: Models with complete and incomplete information. IEEE Transactions on Systems, Man, and Cybernetics. 2, 101 (1981).
[3]. Gibbons R. A Primer in Game Theory. Prentice Hall (1992).
[4]. Kilgour D.M. Game-theoretic properties of final-offer arbitration, Group Decision and Negot. 3, 285 (1994).
[5]. Dao-Zhi Zeng. An Amendment to Final-Offer Arbitration. Working paper, Kagawa Univ. (2006).
[6]. Mazalov V.V., Mentcher A.E., and Tokareva J. S. Equilibrium in a discrete arbitration procedure. Scientiae Mathematicae Japonicae. 63(no 3), 325 (2006).
[7]. Мазалов В.В., Менчер А.Э., Токарева Ю.С. О равновесии в модели переговоров с арбитром. Известия АН. Теория и Системы Управления. 3 (2009).
[8]. Mazalov V., Tokareva J. Equilibrium in combined arbitration procedure. Proc. II International Conf. in Game Theory and Applications, Qingdao, China, Sep. 17-19 (2007).
[9]. Mazalov V.V., Tokareva J.S. Bargaining model on the plane. Algorithmic and Computational Theory in Algebra and Languages // RIMS Kokyuroky 1604. Kyoto University (2008).

ВЕСТНИК САНКТ–ПЕТЕРБУРГСКОГО УНИВЕРСИТЕТА

Серия 8
МЕНЕДЖМЕНТ

Ответственный редактор серии:
Ю.Е. Благов

Издательство Высшей школы менеджмента Санкт-Петербургского университета

Information Sets in the Discrete Differential Search Game with a Team of Pursuers or Evaders

Semyon Mestnikov

Yakutsk State University,
Russian Federation
mestsv@mail.ru

Keywords: *Differential search game, Information set, Mixed strategies, Information set, Probability of detection*

Abstract: *The zero-sum differential search game with discrete moments of detections is considered. Player P tries to maximize the probability of detection of the evader E. Mixed strategies of the pursuer is determined using the auxiliary game with a team of pursuers. The mixed strategies of the evader is determined with help of the an auxiliary game with a team of evaders. For the game with a team of pursuers or evader we construct the information set for the position of the evader.*

Introduction

The zero-sum differential search game between pursuer P and evader E in the class of mixed strategies with finite spectrum is considered([1]; [2]; [3]). Each player has only a priory information on the initial position of the other. The evader is considered detected if he is found in the disk of radius l centered at the position of the pursuer. We assume that the moments of detection are discrete ([6]). Player P tries to maximize (player E tries to minimize) the probability of detection of the evader using mixed strategies. The mixed strategies of the pursuer (evader) determined with help of the an auxiliary game with a team of pursuers (evaders). In this paper for the game with a team of pursuers (evader) we construct the information set for the position of the evader and study properties and approximation problems of this information set. In articles ([4]; [5]) differential search games with continuous moments of detections are investigated.

The dynamics of the game and information

The dynamic of the game is described by the following differential equation

$$P : \dot{x} = f(t,x,u),\ u \in U,\ x_0 \in X_0 \subset R^n,$$

$$E : \dot{y} = g(t,y,v),\ v \in V,\ y_0 \in Y_0 \subset R^n,$$

where U (V) is a compact set in the Euclidean space R^p (R^q), X_0 (Y_0) is a compact set, $t \in [t_0, \infty)$ ([3]).

The detection set S(x) of player P is the disk of radius l centered at the position of the pursuer.

We suppose that player P detects a player E in the discrete times $\sigma = \{t_0 < t_1 < \ldots < t_\gamma < \ldots\}$ where t_γ is a ordinal, $\sup_{\gamma<\delta} = t_\delta \in \sigma$. Introduce the function $F_1^\sigma(x(t), y(t))$ that is defined at the states $x(t)$ and $y(t)$ as follows, $t \in \sigma$:

$$F_1^\sigma(x(t), y(t)) = \begin{cases} 1, & if \;\; y(t) \in S(x(t)), \\ 0, & otherwise. \end{cases}$$

We define the following functional on trajectories $x(\cdot)$ and $y(\cdot)$

$$F_2^\sigma(x(\cdot), y(\cdot)) = \max_{t \in [t_0, \infty)} F_1^\sigma(x(t), y(t)),$$

$$F_2^{\sigma,\tau}(x(\cdot), y(\cdot)) = \max_{t \in [t_0, \tau]} F_1^\sigma(x(t), y(t)), \tau \geq t_0.$$

Information sets in a discrete differential search game with a team of pursuers

We consider an auxiliary game between evader E and some team $\bar{P} = \{P_1, \ldots, P_k\}$ of similar pursuers acting as one. We say that the a strategy $\bar{P}$ of team guarantees k_*-detection at least k_*, $k_* \leq k$ pursuers detects of evader.

For the the fixed strategy $\bar{a} = ((x_0, u_1, \ldots, x_k))$ of team $\bar{P}$, the information set $\Omega_\sigma^{k,k_*}(t) = \Omega_\sigma^{k,k_*}(t)(t, t_0, (x_0^1, \ldots, x_0^k), u_1, \ldots . u_k)$ at time $t \geq t_0$ is defined to be set

$$\Omega_\sigma^{k,k_*}(t) = \{y \in R^n \mid \exists y(t) = y \in Y(t, t_0, y_0), F_3^{\sigma,\tau}((x_1(\cdot), \ldots, x_k(\cdot)), y(\cdot)) < k_*\}.$$

where $Y(t, t_0, y_0)$ is the attainability set of player E at time $t \in [t_0, \infty)$ and

$$F_3^{\sigma,\tau}((x_1(\cdot), \ldots, x_k(\cdot)), y(\cdot)) = \sum_{i=1}^{k} F_2^{\sigma,\tau}(x_i(\cdot), y(\cdot)).$$

Let the information set $\Omega_\sigma^{k,k_*}(t)$ is empty for the auxiliary game with a team $\bar{P}$ there are k pursuers. Then a mixed strategy $\mu = (\mu_1, \ldots, \mu_k)$, $\mu_i = 1/k$, $i = 1, \ldots, k$ of player P given on $\{(x_0^1, u_1(\cdot)), \ldots, (x_0^k, u_k(\cdot)\}$ guarantees detection probability

$$\nu \geq \frac{k_*}{k}$$

Using the principe of transfinite induction (as in article (Mestnikov, 1995)) we construct the approximation set for this information set $\Omega_{\sigma}^{k,k_*}(t)$. In the case that the detection set S(x) is open it is proved that the approximation set and the information set $\Omega_{\sigma}^{k,k_*}(t)$ is equal.

Information sets in a discrete differential search game with a team of evaders

We consider an auxiliary game between pursuer P and some team $\bar{E} = \{E_1,\dots,E_m\}$ of similar evaders acting as one. We say that the a strategy $\bar{b}$ of team guarantees m_* -evasion at least for m_* evaders of team to avoid capture.

The information set $G_{\sigma}^{m,m_*}(t)$ at time $t \in [t_0,\infty)$ is defined to as follows

$$G_{\sigma}^{m,m_*}(t,t_0,Y_0) = \{y \in R^n \mid \exists y_i(t) = y_i(t,t_0,y_0^i,v_i) \in Y(t,t_0,y_0^i),\, y_0^i \in Y_0, i = 1,\dots,m$$

$$\forall x(\cdot)\, (F_3^{\sigma,t}(x(\cdot),(y_1(\cdot),\dots,y_m(\cdot))) \le m - m_*)\},$$

where $Y(t,t_0,y_0)$ is the attainability set of player E at the time $t \in [t_0,\infty)$ and

$$F_3^{\sigma,\tau}(x(\cdot),(y_1(\cdot),\dots,y_m(\cdot))) = \sum_{i=1}^{m} F_2^{\sigma,\tau}(x(\cdot),y_i(\cdot)).$$

Let the information set $G^{m,m_*}(t)$ is not empty for the auxiliary game with a team $\bar{E}$ there are m evaders . Then a mixed strategy $\nu = (\nu_1,\dots,\nu_m)$, $\nu_i = 1/m$, $i = 1,\dots,m$ of player E given on $\{(y_0^1,v_1(\cdot)),\dots,(y_0^m,v_m(\cdot)\}$ guarantees detection probability $\nu \le \frac{m - m_*}{m}$.

We construct the approximation set for this information set $\Omega_{\sigma}^{k,k_*}(t)$ using the principe of transfinite induction (as in article (Mestnikov, 1995)). In the case that the detection set S(x) is open it is proved that the approximation set and the information set $G^{m,m_*}(t)$ is equal.

References

[1] Petrosjan L.A., Zenkevich N.A., (1987). Optimal Search in Conflict Conditions (Russian), Leningrad Univ., Leningrad.

[2] Petrosjan L. A., Garnaev A.Yu. (1992) Search games.(Russian) St.-Peterbg. Gos. Univ., St. Petersburg.

[3] Zenkevich N.A., Mestnikov S.V. (1991) Dynamical search of a moving object in conflict condition. (Russian) Leningrad Univ., Voprosy Mekh. Protsess. Upravl., No. 14, 68 –76
[4] Mestnikov S.V. (1992) Approximation of the domain of indeterminacy in differential search games. (Russian) Differentsialnye Uravneniya 28(6), 967 –972; translation in Differential Equations 28 (1992), no. 6, 774 –778
[5] Mestnikov S.V. (2002) Approximation of the information set in a differential search game with a team of pursuers. (English) Petrosjan, L. A. (ed.) et al., 10th international symposium on dynamic games and applications. St. Petersburg, Russia. In 2 vol. St. Petersburg: International Society of Dynamic Games, St. Petersburg State Univ.. 630-631
[6] Mestnikov S.V. (2005) Information sets in a differential search game with discrete moments of detection. (Russian) Mat. Zametki YaGU, 12, No 2 , 28 –35

ВЕСТНИК САНКТ–ПЕТЕРБУРГСКОГО УНИВЕРСИТЕТА

Серия 10
ПРИКЛАДНАЯ МАТЕМАТИКА, ИНФОРМАТИКА, ПРОЦЕССЫ УПРАВЛЕНИЯ

Издательство Санкт-Петербургского университета

New Coalition Values without Dummy Axiom

George Mironenko[1], **Polina Provotorova**[2] and **Alexandra Zinchenko**[3]

Russian Federation
[1]*georim89@mail.ru*
[2]*prov-pa@inbox.ru*
[3]*Zinch46@mail.ru*

Keywords:*Coalition structure, Coalition value, Consensus value, Shapley value, Axiomatization*

Abstract: *Two extensions of consensus value to CS-games in spirit of Owen value are introduced. The first CS-value arises from applying the consensus value twice: in game between coalitions as well as within coalitions. In contrast, the second CS-value uses the Shapley value in games within coalitions. We proved the axiomatic characterization of new solution concepts.*

We study a CS-game (N,ν,C) where (N,ν) is a TU game and $C = \{C_1,\ldots,C_m\}$ is a coalition structure. Most known CS-values satisfy the dummy player property: if $\nu(S \cup i) = \nu(S)$ for all $S \subseteq N \setminus i$ then player i (dummy) gets nothing even though its coalition is in very strong position. However, for some real situations the dummy axiom does not seem good (van den Brink, 2007). Despite the existence efficient CS-values without dummy player property, they are not reflect the outside options of players within the same structural coalition $C_p \in C$ (Kamijo, 2007). For a construction of new CS-values we use the consensus value $K(N,\nu)$ of game (N,ν) (Ju et al., 2007) being the average of the Shapley value $Sh(N,\nu)$ and the equal surplus solution $E(N,\nu)$, which correspond to polar principles of distributive justice (utilitarian and egalitarian), i.e.

$$K(N,\nu) = \frac{Sh(N,\nu) + E(N,\nu)}{2}. \tag{1}$$

Let $S \subset C_p \in C$. Consider three types of TU games: the reduced game (C_p,ν_p) within coalition C_p, the game (M,ν_C) between components of C and the game $(M,\nu_{C_p^S})$ between components of $C_p^S = \{C_1,\ldots,C_{p-1},S,C_{p+1},\ldots,C_m\}$ were

$$\begin{gathered} M = \{1,\ldots,m\},\ \nu_p(S) = K_p(M,\nu_{C_p^S}), \\ \nu_C(Q) = \nu(\bigcup_{e\in Q} C_e)\ for\ all\ Q \subseteq M, \end{gathered} \tag{2}$$

$$\nu_{C_p^S}(Q) = \nu(S \cup \bigcup_{e \in Q \setminus p} C_e) \ for\ Q \ni p \quad and$$
$$\nu_{C_p^S}(Q) = \nu_C(Q) \ for\ Q \subseteq M \setminus p. \tag{3}$$

Given $i \in C_p \in C$ we define the *consensus CS-value* $KK(N,\nu,C)$ and the *consensus-Shapley CS-value* $KSh(N,\nu,C)$ of game (N,ν,C) by

$$KK_i(N,\nu,C) = K_i(C_p,\nu_p), \ \ KSh_i(N,\nu,C) = Sh_i(C_p,\nu_p) \,. \tag{4}$$

By means of (1)-(4) and the formulas for $Sh(N,\nu)$ and $E(N,\nu)$ we can express new CS-values through ν. Hence, these values can be easy computed. Let $Du(N,\nu)$ be the set of all dummy players in game (N,ν) and $F(N,\nu,C)$ be a CS-value. For the characterization of $KK(N,\nu,C)$ and $KSh(N,\nu,C)$ we use four standard axioms (efficiency (**E**), additivity (**A**), external symmetry (**ES**), internal symmetry (**IS**)) and two modified versions of dummy player property (**MD1**, **MD2**).

MD1. If $i \in Du(C_p,\nu_p) \neq \varnothing$ then $F_i(N,\nu,C) = \dfrac{K_p(M,\nu_C) - \sum_{j \in C_p} K_p(M,\nu_{C_p^j})}{2\,|\,C_p\,|}$. If $C_p \subseteq Du(N,\nu)$ then $F_i(N,\nu,C) = \dfrac{K_p(M,\,\nu_C)}{|\,C_p\,|}$.

MD2. If $i \in Du(C_p,\nu_p) \neq \varnothing$ then $F_i(N,\nu,C) = 0$. If $C_p \subseteq Du(N,\nu)$ then $F_i(N,\nu,C) = \dfrac{K_p(M,\,\nu_C)}{|\,C_p\,|}$.

Unlike a dummy axiom, **MD1** and **MD2** define a payoffs not all dummy players in game (N,ν). Moreover, these payoffs can be nonzero. Next lemma show that dummy player in (N,ν) not always remains dummy in game within his coalition. Let $i \in Du(N,\nu) \cap C_p \neq \varnothing$. Then $i \in Du(C_p,\nu_p)$ iff $\nu(N \setminus C_p) = \sum_{e \in M \setminus p} \nu(C_e)$. The consensus CS-value is the unique value on the class of CS-games satisfying **E**, **A**, **ES**, **IS** and **MD1**. The consensus-Shapley CS-value is the unique value on the class of CS-games satisfying **E**, **A**, **ES**, **IS** and **MD2**. De Waegenaere et al., (2007, Example 5) consider the situation with the investor set $N = \{1,2,3\}$, the capital endowments 60,40,40 and the investment opportunity set which contains: a 10 percent bank deposit and a production process that requires an initial investment of 100 and yields a 20 percent rate of return. This situation is modelled by game (N,ν) where

$$\begin{aligned}&\nu(1)=66,\, \nu(2)=\nu(3)=44,\, \nu(1,2)=\nu(1,3)=120,\\ &\nu(2,3)=88,\, \nu(N)=164\end{aligned}.$$

For clarity, shall go to zero-normalized game:

$$\nu'(i)=0,\, i\in N,\, \nu'(2,3)=0,\, \nu'(1,2)=\nu'(1,3)=\nu'(N)=10\,.$$

The unique core element $(10,0,0)$ gives the worth of grand coalition to the first investor, ignoring the productive role of other players. $K(N,\nu')=(5,2.5,2.5)$. To show a difference between new CS-values and the collective value $Sh^{\omega}Sh^{\omega}(N,\nu',C)$ as well as the two-step Shapley value $ShSh(N,\nu',C)$ (Kamijo, 2007), consider the structure $C=\{\{1,2\},\{3\}\}$. We have $KK(N,\nu',C)=KSh(N,\nu',C)=(7.5,2.5,0)$ and $ShSh(N,\nu',C)=Sh^{\omega}Sh^{\omega}(N,\nu',C)=(5,5,0)$. Thus, it is difficult to disagree that for given game the solution concepts based on the consensus value prescribes a rather natural and desired outcomes.

References

[1] Van den Brink, R. (2007). Null or nullifying players: The difference between the Shapley value and equal division solutions. Journal of Economic Theory, 136, 767-775.
[2] Ju, Y., P. Borm and P. Ruys (2007). The consensus value: a new solution concept for cooperative games. Social Choice Welfare, 28(4), 685 –703.
[3] Kamijo, Y. (2007). A collective value: a new interpretation of a value and a coalition structure. 21COE-GLOPE Working Paper Series, 27, 1 –23.
[4] de Waegenaere, A., J. Suijs and S. Tijs (2005). Stable profit sharing in cooperative investments. OR Spectrum, 27(1), 85 –93.

Completeness on Simple Games

Xavier Molinero[1], **Josep Freixas**[2], **Martin Olsen**[3] and **Maria Serna**[4]

[1,2]*Escola Politècnica Superior d'Enginyeria de Manresa, Spain*
[1]*molinero@lsi.upc.edu*
[2]*josep.freixas@upc.edu*
[3]*University of Aarhus, Denmark*
mo@cs.au.dk
[4]*Universitat Politècnica de Catalunya, Spain*
mjserna@lsi.upc.edu

Keywords: *Simple Games, Weighted Games, NP-Completeness*

Simple games cover voting systems in which a single alternative, such as a bill or an amendment, is pitted against the status quo. A simple game or a yes-no voting system is a set of rules that specifies exactly which collections of ''yea'' votes yield passage of the issue at hand. A collection of ''yea'' voters forms a winning coalition.

In this work we perform a complexity analysis on problems defined on such families of games. In the case that the number of players is large, as pointed out in E.Elkind et al., from a computational point of view, the key issues relating to coalitional games are, first, how such games should be represented (since the obvious representation is exponentially large in the number of players); and second, the extent to which cooperative solution concepts can be efficiently computed. We undertake here this task. This analysis as usual depends on the game representation used as input. We consider four natural explicit integer representations: winning, losing, minimal winning, and maximal losing.

We analyze the complexity of testing whether a game is simple and testing whether a game is weighted. It is showed that, for the four types of representations, both problems can be solved in polynomial time. We also provide results on the complexity of testing whether a simple game or a weighted game is of a special type. We analyze strongness, properness, decisiveness and homogeneity, which are desirable properties to be fulfilled for a simple game.

Finally, some considerations on the possibility of representing a game in a more succinct representation are also studied in depth.

Acknowledgements

Josep Freixas was partially supported by Grant MTM 2006-06064 of ‘‘Ministerio de Ciencia y Tecnología y el Fondo Europeo de Desarrollo Regional", MTM 2009-08037 from the Spanish Science and Innovation Ministry, SGR 2009-1029 of ‘‘Generalitat de Catalunya’’ and 9-INCREC-11 of ‘‘(PRE/756/2008) Ajuts a la iniciació/reincorporació a la recerca (Universitat Politècnica de Catalunya)".

Xavier Molinero was partially supported by programme TIN2006-11345 (ALINEX) from the Spanish ‘‘Ministerio de Educación y Ciencia", ALBCOM-SGR 2009-1137 of ‘‘Generalitat de Catalunya’’, and 9-INCREC-11 of ‘‘(PRE/756/2008) Ajuts a la iniciació/reincorporació a la recerca (Universitat Politècnica de Catalunya)".

Maria Serna was partially supported by FET pro-active Integrated Project 15964 (AEOLUS), the Spanish projects TIN-2007-66523 (FORMALISM), TIN-2005-09198-C02-02 (ASCE), and TIN2005-25859-E, and ALBCOM-SGR 2009-1137 of ‘‘Generalitat de Catalunya’’.

We would also like to acknowledge Gerth S. Brodal from University of Aarhus for valuable comments and constructive criticism.

References

[1] F.Carreras and J.Freixas. Complete simple games. Mathematical Social Sciences, 32:139-155, 1996.

[2] X.Deng and C.H.Papadimitriou. On the complexity of cooperative solution concepts. Math. Oper. Res., 19(2):257-266, 1994.

[3] E.Elkind, L.A.Goldberg, P.W.Goldberg, and M.Wooldridge. Computational complexity of weighted threshold games. In Proceedings of the Twenty-Second AAAI Conference on Artificial Intelligence, July 22-26, 2007, Vancouver, British Columbia,Canada, pages 718-723, 2007.

[4] M.R.Garey and D.S.Johnson. Computers and Intractability: A Guide to the Theory of NP-Completness. W.H.Freeman, San Francisco, New York, USA, 1979.

[5] N.Karmarkar. A new polynomial-time algorithm for linear programming. Combinatorica, 4:373-395, 1984.

[6] Y.Matsui and T.Matsui. NP-completeness for calculating power indices of weighted majority games. Theoretical Computer Science, 263(1-2):305-310, 2001.

[7] D.Mehta and V.Raghavan. Decision tree approximations of boolean functions. Theoretical Computer Science, 270(2):609-623, 2002.

[8] C.H.Papadimitriou. Computational Complexity. Addison Wesley, 1994.

[9] K.Prasad and J.S. Kelly. NP-completeness of some problems concerning voting games. International Journal of Game Theory, 19(1):1-9, March 1990.

[10] J.Rosenmüller. An algorithm for the construction of homogeneous games. In Opitz/Rauhut (Hrsg.), editor, Ökonomie und Mathematik, pages 63-74. Springer-Verlag, 1987.
[11] A.D.Taylor and W.S.Zwicker. Simple games: desirability relations, trading, and pseudoweightings. Princeton University Press, New Jersey, USA, 1999.

МАТЕМАТИЧЕСКАЯ ТЕОРИЯ ИГР И ЕЕ ПРИЛОЖЕНИЯ

Ответственный редактор:
Л.А. Петросян
Зам. ответственного редактора:
В.В. Мазалов
Ответственный секретарь:
Н.А. Зенкевич

Выходит ежеквартально с января 2009 года

Non-cooperative Games with Confirmed Proposals

Aldo Montesano[1], **Giuseppe Attanasi**[2] and **Nicolaos Georgantzis**[3]

[1]*Bocconi University,*
Italy
aldo.montesano@unibocconi.it

[2]*University of Toulouse,*
France
attanasi@cict.fr

[3]*Universidad de Granada,*
Spain
ngeorgantzis@ugr.es

Keywords: *Confirmed proposals, Bargaining games, Cooperative results*

There is a huge literature on bargaining games as non-cooperative games. In some of these articles, the interactions between cooperative and non-cooperative games emerge. However, traditional game theorists usually start their analysis by exogenously specifying that the strategic interaction among players belongs to the family of non-cooperative games or to the one of cooperative games. However, in real life strategic interactions, the nature of the game itself is an endogenous feature and it often happens that players start to act competitively, realizing only ex-post that they were playing a 'cooperative game', because the outcome they reach is always more collusive than what they imagined at the beginning. The 'confirmed proposal' mechanism we introduce in this paper is a non-cooperative process leading to cooperative results. Through our bargaining process, we analyze how players could 'switch' from a non-cooperative to a cooperative game only by 'bargaining' with their opponents, without changing the rules of the strategic interaction in which they are involved. The paper is both theoretical and experimental.

The theoretical part of the paper concerns the construction of optimal contracts in simple two players dynamic bargaining games, in which at least one player, in order to let the game end, must confirm her strategy once known the strategy chosen by her opponent. We define this family of games as Games with Confirmed Proposals (GCP henceforth). These interactive strategic situations can be games with perfect/imperfect information and/or with complete/incomplete information. When information is incomplete, players can exploit the bargaining process by extracting information through their opponent's proposals. We use a simple two players static or dynamic game (we call it constituent game) to 'describe' the payoffs of the bargaining game built on it. There is a constituent game (whose execution leads to the players' final payoffs) and a dynamic game whose actions in each bargaining period are 'proposals' of the actions specified in the constituent game. Notice that even though we deal with constituent games whose players' strategy space is finite, in the supergame (bargaining game) built on them each player's strategy space is infinite and the supergame itself is infinite (even though the constituent game is finite). We call 'confirmed agreement' the related equilibrium contract (on the action profiles to play) signed by the players. In the paper, we show that, under mild assumptions, the equilibrium outcome of the bargaining process is unique. Nonetheless, we show that in social dilemma games (as the prisoner's dilemma) applying the "confirmed proposal" rules leads to the cooperative and Pareto-efficient outcome with respect to the standard one-shot social dilemma game (constituent game).

The experimental part of the paper concerns the laboratory analysis of the GCP mechanism applied to a prisoner's dilemma strategic situation (the prisoner's dilemma is the constituent game). More precisely, we have three different treatments:
(a) Almost Infinitely Repeated Game: the one-shot prisoner's dilemma is played an unknown number of periods (with choices being observable after each period), with the game randomly ending after each period (end rate equal to 2%);
(b) Symmetric Confirmed Proposals: in each pair of subjects one of them (the "proposer") proposes his action (cooperate or defect) and waits to know the action (cooperate or defect) proposed by his counterpart (the "responder") who has previously seen his proposal. Then, the proposer can confirm or not the action profile. In case he does not, the two roles of proposer and responder are switched and the game restarts. The game ends only after a proposer has confirmed his proposal, once seen that of the responder.

(c) Asymmetric Confirmed Proposals: in each pair of subjects only one of them can propose his action and wait to know the action proposed by his counterpart, before confirming (or not) the action proposed. In case he does not, the game restarts and he continues to play as a proposer. Again, the game ends only after the proposer has confirmed his proposal, once seen that of the responder. In this treatment the "power of confirmation" is asymmetric.

In each of the three treatments, the payoffs of the bargaining game are those of the prisoner's dilemma. We know from game theory that the most cooperative framework built on a prisoner's dilemma is the one in which the game is repeated an infinite number of times. In fact, the zero-discounting, infinitely repeated version of the prisoner's dilemma has a subgame perfect equilibrium leading to cooperation in each repetition.

From the side of games with confirmed proposals, we prove theoretically that the power of confirmation does play a role: when only one of the two players plays as proposer in each bargaining stage (asymmetric confirmed proposals), she can reach the asymmetric confirmed proposal which is advantageous for her. When, instead, the power of confirmation is symmetric (symmetric confirmed proposals), it is not possible to reach an asymmetric agreement in equilibrium. Moreover, when the power of confirmation is asymmetric, the Nash Equilibrium outcome of the one-shot prisoner's dilemma is still a subgame perfect equilibrium outcome of the game, while when players alternate in exerting the power to end the game, the non-cooperative outcome is not a subgame perfect equilibrium: all the subgame perfect equilibria lead to the cooperative (symmetric) confirmed agreement. Therefore, despite the fact that the constituent prisoner's dilemma has a unique Nash non-cooperative equilibrium, the confirmed proposal version always includes the cooperative Pareto-superior outcome among its subgame perfect equilibria, independently of the allocation of the power of confirmation. The non-cooperative outcome is not an equilibrium when the power of confirmation is symmetric. In that case, the Pareto-optimal symmetric outcome is the only subgame perfect equilibrium outcome of the game. The only confirmed proposal structure allowing the proposer to gain from his power of confirmation is the asymmetric one. Only in this game the asymmetric outcome involving the highest payoff for the propose is a subgame perfect equilibrium.

Therefore, our first research question is: Does the GCP version of social dilemma games, lead unambiguously to the cooperative outcome? To empirically evaluate the success of our framework as a cooperation inducing device, we compare it to the well known collusion facilitating environment of an Indefinitely (almost infinitely) Repeated prisoner's dilemma Game (treatment (b) and (c) vs treatment (a)).

Our second research question is: Can a GCP with asymmetric power of confirmation as easily as predicted lead to symmetric confirmed agreements? Alternatively, we want to test whether this bargaining structure leads to a less cooperative behavior with respect to the case in which both players can alternatively make a proposal and confirm it (treatment (c) vs treatment (b)). Finally, we are interested in GCP as a form of communication between players. Through the bargaining structure developed, we believe that it could be understood how players communicate, without using more 'explicit' communication devices. Moreover, we could shed some light on what a subject wants to communicate to another, on what she is able to communicate as well as to understand from the proposals received.

Our experimental results show that the GCP structure applied to social dilemma games lead to cooperation more than do other cooperation-enhancing mechanisms, like the corresponding indefinitely repeated game with a tiny (2%) end-game probability.

Moreover, in the prisoner's dilemma modelled as a Game with Confirmed Proposals, the existence of asymmetric power of confirmation does not significantly affect the frequency of cooperation. On the contrary, the existence of an exogenous leader increases the likelihood of immediate cooperation, although it entails some risk of very long negotiation games.

Finally, our experimental results show how Games with Confirmed Proposals can be used to create a glossary of bargaining semantics for tacit communication among agents concerning their rationality/irrationality, patience/impatience, social and psychological preferences, etc. through the signals contained in their proposals and confirmation strategies.

On a Condition Measure for Matrix Game Problem and its Relation to Metric Regularity Modulus

Boris Mordukhovich[1], **Javier Peña**[2] and **Vera Roshchina**[3]

[1]*Distinguished University*
United States
boris@math.wayne.edu

[2]*Associate Professor of Operations*
United States
boris@math.wayne.edu

[3]*vera.roshchina@gmail.com*

Keywords: *Condition measure for matrix game problem, Nesterov smoothing algorithm, Metric regularity modulus*

Abstract: *The condition measure k(A) used in [Gilpin, A., Pena, J., and Sandholm, T. First-order algorithm with O(ln(1/ε)) convergence for ε-equilibrium in two-person zero-sum games. In Proceedings of AAAI'2008 (2008), pp. 75–82] to estimate the complexity of a new algorithm for solving two-person zero-sum matrix game is evaluated in this work. In addition to giving an exact expression for k(A), we show that this condition measure is closely related to the metric regularity modulus of an associated mapping.*

In [2] a new condition measure was introduced to evaluate the complexity of a first-order algorithm for solving a two-person zero-sum game

$$\min_{x\in Q_1}\max_{y\in Q_2} x^{\mathrm{T}}Ay = \max_{y\in Q_2}\min_{x\in Q_1} x^{\mathrm{T}}Ay, \tag{1}$$

where $A \in \mathbb{R}^{m\times n}$, and each of the sets Q_1 and Q_2 is either a simplex (in a matrix game formulation) or a more elaborate polytope (in the case of sequential games). It was shown in [2] that an iterative version of Nesterov's first-order smoothing algorithm [5] computes an ε-equilibrium point of (1) in $O(\|A\|k(A)\ln(1/\varepsilon))$ iterations, where $k(A)$ is a condition parameter that depends only on the matrix A. The dependence of this complexity bound on ε is exponentially better than the complexity bound $O(1/\varepsilon)$ of the original Nesterov's algorithm. The authors in [2] showed that $k(A)$ is always finite but their proof is non-constructive. In particular, no explicit upper bound on $k(A)$ was given.

We derive two major results related to the computation of the condition measure $k(A)$. First, we provide an explicit expression for $k(A)$ in terms of the solution to (1). This is a major step towards performing further complexity analysis of the algorithm [2]. Second, we show that this condition measure equals the metric regularity modulus of a related functional. The latter result is not exactly unexpected, as recently there have been works relating metric regularity to complexity estimates of numerical algorithms (e.g. see [1,4]).

Following [2], we consider a two-person zero-sum matrix game equilibrium problem:

$$\min_{x\in\Delta_m}\max_{y\in\Delta_n} x^{\mathrm{T}}Ay = \max_{y\in\Delta_n}\min_{x\in\Delta_m} x^{\mathrm{T}}Ay, \tag{2}$$

where $\Delta_m := \{x \in \mathrm{IR}^m | \sum_{i=1}^m x_i = 1, x \geq 0\}$ is the set of mixed strategies for a player with m pure strategies. That is, if player 1 plays $x \in \Delta_m$ and player 2 plays $y \in \Delta_n$, then 1 gets payoff $-x^T Ay$ and 2 gets payoff $x^T Ay$. This problem can be cast as as a nonsmooth convex optimization problem

$$\min_{(x,y)\subseteq\Delta_m\times\Delta_n} F(x,y)$$

where

$$F(x,y) = \max\{x^T Av - u^T Ay \mid (u,v) \in \Delta_m \times \Delta_n\} \tag{3}$$

Observe (see [2]) that $\min\{F(x,y) \mid (x,y) \in \Delta_m \times \Delta_n\} = 0$. A pair (x,y) is in a Nash equilibrium if $F(x,y) = 0$, and it is in an ε-Nash equilibrium if $F(x,y) \leq \varepsilon$. Let S be the set of exact solutions to the equilibrium problem (2):

$$S := \{(x,y) \in \Delta_m \times \Delta_n | F(x) = 0\} = F^{-1}(0) \cap (\Delta_m \times \Delta_n).$$

The *condition measure* $k(A)$ of the matrix A is defined as follows:

$$k(A) = \inf\left\{k \mid dist((x,y),S) \leq kF(x,y) \quad \forall (x,y) \in \Delta_m \times \Delta_n\right\}, \tag{4}$$

where

$$dist((x,y);S) = \min\{\|(x,y)-(u,v)\| \,|(u,v) \in S\},$$

the norm $\|\cdot\|$ is Euclidean: $\|w\| = \sqrt{w^{\mathrm{T}}w}$, and by (u,v) we denote a column-vector composed of u and v.

Our main result establishes a characterization of $k(A)$ via the metric regularity modulus of a related mapping Φ defined below. This in turn yields an exact formula for evaluating $dw_p / dt \leq G_p(t, x, w_p),$. Before formally stating our result, we need to introduce some notation.

Let $\Phi : \mathbb{R}^{m+n} \to \mathbb{R}$ be a multifunction defined as follows:

$$\Phi(x,y) = \begin{cases} [F(x,y), +\infty), & (x,y) \in \Delta_m \times \Delta_n, \\ \varnothing, & \text{otherwise.} \end{cases}$$

Let $a_i,\ i = 1,\ldots,n,$ and $-b_k^T$, $k = 1,\ldots,m$ be the columns and the rows of A respectively. Let $e_j, j = 1,\ldots,m+n$ denote the unit vectors in IR^{m+n}, that is, $(e_j)_\ell = 0\ \forall \ell \neq j, (e_j)_j = 1.$ For a positive integer p, let $1_p := [1 \ \ldots \ 1] \in \mathbb{R}^p$. Finally, for a given point $(x,y) \in \Delta_n \times \Delta_m$ let

$$I(x) := \{\bar{i} \in 1:n \mid a_{\bar{i}}^{\mathrm{T}} x = \max_{i \in 1:n} a_i^{\mathrm{T}} x\}, \quad K(y) := \{\bar{k} \in 1:m \mid b_{\bar{k}}^{\mathrm{T}} y = \max_{k \in 1:m} b_k^{\mathrm{T}} y\},$$

and

$$J(x,y) = \{j \mid x_j = 0\} \cup \{j = m+p \mid y_p = 0\}.$$

Theorem 1 Let F be defined by (3) and $k(A)$ be defined by (4), and assume $\Delta_m \times \Delta_n \setminus S \neq \varnothing$. Then

$$k(A) = \sup_{(x,y) \in \Delta_m \times \Delta_n \setminus S} \operatorname{reg} \Phi((x,y) \mid F(x,y)),$$

where $\operatorname{reg} \Phi((x,y) \mid F(x,y))$ is the metric regularity modulus of Φ at (x,y) for $F(x,y)$.

For every $(x,y) \in \Delta_m \times \Delta_n \setminus S$ the quantity $\operatorname{reg} \Phi((x,y) \mid F(x,y))$ can be evaluated exactly:

$$\operatorname{reg} \Phi((x,y) \mid F(x,y)) = \frac{1}{\operatorname{dist}(0, \partial F(x,y) + N_{\Delta_m \times \Delta_n}(x,y))},$$

where $\partial F(x,y)$ is the (convex) subdifferential of F at (x,y), and $N_{\Delta_m \times \Delta_n}(x,y)$ is the convex normal cone to $\Delta_m \times \Delta_n$ at (x,y).

The subdifferential $\partial F(x,y)$ and the normal cone $N_{\Delta_m \times \Delta_n}(x,y)$ can be evaluated exactly:

$$\partial F(x,y) = \mathrm{co}\{(a_i,b_k) \mid i \in I(x), k \in K(y)\},$$

$$N_{\Delta_m \times \Delta_n}(x,y) = \mathrm{span}\{1_m\} \times \mathrm{span}\{1_n\} - \mathrm{cone}\ \mathrm{co}\{e_j \mid j \in J(x,y)\}.$$

Theorem 1 readily yields the following characterization for the quantity $k(A)$:

Corollary 2 The condition number $k(A)$ defined in (4) is equal to

$$\sup_{(x,y)\in\Delta_m\times\Delta_n\setminus S} \frac{1}{\mathrm{dist}(0, \mathrm{co}\{(a_i,b_k) \mid i\in I(x), k\in K(y)\} + \mathrm{span}\{1_m\}\times\mathrm{span}\{1_n\} - \mathrm{cone}\ \mathrm{co}\{e_j \mid j\in J(x,y)\})} \tag{5}$$

The proof of Theorem 1 relies on variational analysis techniques. However, Corollary 2 can be proved using convex optimization techniques. The latter is direct and uses simpler tools but it is somewhat more laborious and does not provide the connection between $k(A)$ and the metric regularity of the mapping Φ.

Observe that evaluating the expression (5) is at least as hard as solving (2), as we need to know the optimal set S in advance. This agrees with the phenomenon well-known in complexity theory: evaluating condition number is in general not easier than solving the original problem. For example, this statement is true for such fundamental complexity measures as the condition number of a matrix (see [3]) used in estimating complexity of numerical linear algebra algorithms, Renegar's condition number (see [6,7]) which characterizes difficulty of solving conic feasibility problems, and the ‘‘measure of condition" for finding zeros of complex polynomials inroduced by Shub and Smale (see [8]).

In view of the aforesaid, it is highly unlikely that there exists a straightforward way to compute our condition measure, so the purpose of this work is not obtainig an easily computable expression for $k(A)$, but rather gaining a better understanding on how exactly the problem data influences the condition measure. Observe that the expression

(5) is much easier to evaluate and analyse than the original expression (4). This is valuable for the subsequent average-case or smoothed analysis of the algorithm, singling out classes of well-conditioning problems, and even making further improvements to the algorithm (such as preconditioning).

References

[1] Aragón Artacho, F. J., Dontchev, A. L., and Geoffroy, M. H. Convergence of the proximal point method for metrically regular mappings. ESAIM Proceedings 17 (2007), 1–8.

[2] Gilpin, A., Pena, J., and Sandholm, T. First-order algorithm with $O(\ln(1/\varepsilon))$ convergence for ε-equilibrium in two-person zero-sum games. In Proceedings of AAAI'2008 (2008), pp. 75–82.

[3] Higham, N. Accuracy and Stability of Numerical Algorithms. SIAM, 1996.

[4] Lewis, A. S., Luke, D. R., and Malick, J. Local linear convergence for alternating and averaged nonconvex projections. Foundations of Computational Mathematics 9, 4 (June 2009), 485–513.

[5] Nesterov, Y. Smooth minimization of nonsmooth functions. Math. Progr. 103 (2005), 127–152.

[6] Peňa, J., and Renegar, J. Computing approximate solutions for conic systems of constraints. Math. Progr. 87 (2000), 351–383.

[7] Renegar, J. Linear programming, complexity theory and elementary functional analysis. Math. Progr. 70 (1995), 279–351.

[8] Shub, M., and Smale, S. Complexity of Bézout's Theorem I: Geometric Aspects. Journal of the American Mathematical Society 6, 2 (April 1993), 459– 501.

On a Class of Games with a Polyhedral Set of Nash Equilibria

Pierre von Mouche

Wageningen Universiteit
Netherlands
pvmouche@gmx.net

Keywords: *Cournot oligopoly, Transboundary pollution, Non-differentiable payoff function, Aggregative game, Non-uniqueness of Nash equilibria*

A class of games in strategic form with a polyhedral set of Nash equilibria is identified. There is special attention to conditions that guarantee that there is at most one Nash equilibrium, and to the relation of this property with differentiability of payoff functions. The power of the results is illustrated by applying them to homogeneous Cournot oligopoly games and to transboundary pollution games with uniform pollution. The way of obtaining the results is by a teamed analysis of marginal reductions.

Journals in Game Theory

GAMES AND ECONOMIC BEHAVIOR

Editor
Ehud Kalai

ELSVIER

Stable Group Purchasing Organizations

Mahesh Nagarajan[1], **Greys Sošić**[2] **and Hao Zhang**[3]

[1]*Sauder School of Business, University of British Columbia,*
Vancouver, B.C., Canada V6T 1Z2
mahesh.nagarajan@sauder.ubc.ca,
[2,3]*Marshall School of Business, University of Southern California,*
Los Angeles, California 90089
[2]*sosic@marshall.usc.edu,*
[3]*zhanghao@marshall.usc.edu*

Keywords: *Coalition stability, Group purchasing*

Abstract: *In this paper, we study the stability of Group Purchasing Organizations (GPOs). GPOs exist in several sectors and benefit its members through quantity discounts and negotiation power when dealing with suppliers. However, despite several obvious benefits, GPOs suffer from member dissatisfaction due to unfair allocations of the accrued savings among its members. We first explore the benefits of allocation rules that are commonly reported as being used in practice. We characterize stable coalitional outcomes when these rules are used and provide conditions under which the grand coalition emerges as a tenable outcome. These conditions are somewhat restrictive. We then propose an allocation mechanism based on the marginal value of a member's contribution and find that this leads to stable GPOs in many scenarios of interest. In this analysis, we look at discount schedules that encompass a large class of practical schedules and analyze cases when purchasing requirements of the members are both exogenous as well as endogenous. We use a concept of stability that allows for players to be farsighted, i.e., players will consider the possibility that once they act (say by causing a defection), another coalition may react, and a third coalition might in turn react, and so on, nullifying their original advantage in making the initial move.*

Group purchasing organizations (GPOs) are coalitions of several firms (buyers) who pool their purchasing requirements and buy large quantities of a particular product from a seller. The advantage of belonging to a GPO is evident. GPOs are able to take advantage of significant quantity discounts from the seller and transaction costs may be lowered by bundling different orders. In certain sectors, GPOs have high negotiation power and receive preferred terms of trade.

GPOs, henceforth also referred to as purchasing coalitions, are seen in various industry sectors. The origins of such coalitions can be traced back to the evolution of cooperatives, where cooperatives acted as purchasing and selling coalitions. Today, purchasing coalitions can be seen in virtually every sector spanning health care, education, retail, etc. Despite the touted benefits of forming such organizations, the business and academic literature lament the fact that GPOs do not sustain themselves, often due to disagreements between members. GPOs see membership numbers often fluctuate. Further, problems often arise due to unequal member contributions, with powerful members building barriers to keep out smaller participants from entering the GPO. The very nature of these coalitions causes reasons for conflict. Consider a group of buyers, each with a certain purchasing requirement, who decide to form a coalition. The discount from the seller is based on the aggregated purchasing quantity. Thus, each member of the coalition is able to receive a lower price than what he would have received based solely on his individual order. However, when a set of buyers with heterogeneous requirements form a coalition, it is not immediately clear how the benefits of this lowered price are to be allocated among coalition members. This discord leads to a lack of commitment among coalition members and to potential instability. A comprehensive study of several purchasing consortia in Europe indicates that over a quarter of such coalitions are acutely aware of inherent unfairness in splitting the savings obtained by the coalition. An often-used mechanism that allocates savings internally to the coalition, known as equal price (i.e., one where all members of the coalition pay the same price per item), is an excellent example that opens itself to this criticism. The common wisdom that gains accrued by the coalitions may far surpass any cause for discord is increasingly questioned by coalition members and this may explain the short life of several such consortia. Thus, an important strategic issue to be considered when setting up a GPO seems to be one of how to allocate the gains realized by such a coalition, which is intimately tied to the eventual stability of these coalitions. Despite recognizing the importance of this topic, neither the literatures in operations management that deals with purchasing nor the ones in economics analyze a comprehensive model of purchasing consortia and offer any robust remedies.

In this paper, we examine how allocations among group members in a purchasing coalition should be designed (especially when contributions by individual members may not be equal) and analyze the related issues of stability of purchasing coalitions. We first search the extant literature (both academic and industry) to find a good representation of a quantity discount schedule. The functional form we end up using fits best with what is seen in practice and possesses a few attractive analytical properties as well. In terms of dividing the gains of a coalition, we use and suggest a few allocation rules that make sense from both a practical perspective as well as are related to theoretical concepts from cooperative games: (i) the equal allocation principle, which allocates to each member the same portion of accrued savings; (ii) the quantity-based proportional allocation rule, which allocates the savings proportionally to the buyers' ordering quantities; and (iii) the Shapley value, which tries to compute the marginal contribution of an individual member of the coalition as an allocation rule. The issue of allocations, as mentioned earlier, is tied to the question of stability of the alliance. We use cooperative game theory to analyze the stability of coalitions. A commonly used concept of stability, popular in the operations literature, is the core. The core distinguishes allocation rules that yield a stable alliance of all players, but suffers from the problem of myopia. That is, it precludes the possibility that players and sub-coalitions may consider the possibility that once they act (say, by causing a defection), another coalition may react, and a third coalition might in turn react, and so on, nullifying their original advantage in making the initial move. A concept of stability that takes such a farsighted view of players is the Largest Consistent Set (LCS, Chwe 1994). In our analysis, we allow for players to be farsighted and thus primarily use the LCS to evaluate the stability of coalitions.

Our main findings when considering the aforementioned allocation rules are as follows. We first look at a scenario in which the requirement of each firm (i.e., its order quantity) is exogenous to the discussion and thus independent of the specifics of the discount schedule. Such a scenario is realistic in several public-sector GPOs (the healthcare sector being a good example). Here, purchase orders are periodically determined based on needs, and less on other factors such as profit maximization or re-selling decisions. In this setting, when buyers are homogeneous, the alliance of all buyers (the grand coalition) is stable and sustains itself independent of the allocation rules. This result is not surprising. However, when the buyers are heterogeneous, the Shapley value alone produces fair allocations that ensure the stability of the grand coalition. When either equal allocation rules or quantity-based proportional allocation rules are used and one looks at the farsighted stability of coalitions, there is a strong tendency in which the buyers with large orders split to form their own GPO, leaving buyers with smaller contributions to themselves. This leads to the following insight—if one needs to sustain a GPO with heterogeneous buyers, contracts that allocate savings between buyers need to be carefully arrived at, using, for instance, the Shapley value allocations. Several of the currently followed allocation rules that fail to do this will result in eventual instability, even if they seem fair (for instance, quantity-based proportional allocation) at the outset.

Next, we look at a scenario in which the quantity required by each buyer is endogenous to the model. An illustrative case is that of a profit-maximizing price-setting firm which decides how much its requirements are by considering both the quantity discount schedule offered by the seller and its own downstream price-driven demand. The firm under question simply sets its demand curve and the discount schedule (supply curve) equal to each other and derives its requirements based on this intersection. As one can imagine, this analysis can be algebraically tedious, as solving for the order quantity may not be simple. When we consider linear demand and discount schedules, we are able to show that several of our results and insights from previous analysis with exogenous demands still hold. In particular, we show that it is still advisable to use a Shapley-value-based allocation to guarantee the stability of the GPO. We also show that when quantity decisions are endogenous and one uses either the equal allocation or the quantity based allocation rules, under certain assumptions our earlier result continues to hold (i.e., large coalitions coalesce together and leave the smaller members out). This prediction of ours is seen widely in practice and is also the topic of several studies and reports on purchasing organizations.

We believe our findings above yield important insights to firms that contemplate joining purchasing coalitions, or to intermediaries who wish to create successful and efficient GPOs. Since the creation of GPOs has substantial consequences to social surplus and creates efficiencies in many supply chains, we believe it is particularly important for the relevant players to understand and pursue strategies that will contribute to its success.

Claim Problems with Coalition Demands

Natalia Naumova

St. Petersburg State University,
Faculty of Mathematics and Mechanics
Russia
nataliai.naumova@mail.ru

Keywords: *Claim problem, Proportional method, Uniform losses method, Nucleolus, Weighted entropy*

A claim problem is a triple (N,t,x) where N is a finite set of agents, the positive real number t represents the amount of resources to be divided by agents, the vector $x=\{x_i\}_{i\in N}$ specifies for each agent i a claim $x_i>0$. A solution to the rationing problem is a vector $y=\{y_i\}_{i\in N}$ such that $y_i\geq 0$ for all $i\in N$, $\sum_{i\in N}y_i=t$. A claim method is a map that associates to any claim problem its solution. Claim methods and their axiomatic justifications are described in surveys [1] and [2].

Here we consider a generalized claim problem. For a fixed collection of coalitions of agents $A\subset 2^N$, we replace the vector x (the set of claims of agents) by $\{v(T)\}_{T\in A}$, where $v(T)$ is a claim of T. Thus, a generalized claim problem is a family $(N,A,t,\{v(S)\}_{S\in A})=(N,A,t,v)$. We may consider a generalized claim problem as a TU–cooperative game with limited cooperation possibilities. A generalized claim method is a map that associates to any generalized problem (N,A,t,v) a subset of the set $\{\{y_i\}_{i\in N}: y_i\geq 0,\sum_{i\in N}y_i=t\}$. We assume that A covers N.

In this paper we consider generalizations of two claim methods: the Proportional method and the Uniform Losses method (also called Constrained Equal Losses method).

Let $X\subset R^n$, $f_1,\dots,f_k$ be functions defined on X. For $z\in X$, let π be a permutation of $\{1,\dots,n\}$ such that $f_{\pi(i)}(z)\leq f_{\pi(i+1)}(z)$, $\theta(z)=\{f_{\pi(i)}(z)\}_{i=1}^n$. Then $y\in X$ belongs to the nucleolus with respect to $f_1,\dots,f_k$ on X if

$$\theta(y) \geq_{lex} \theta(z) \quad forall \quad z \in X.$$

For generalizations of Proportional method we suppose that $v(T) > 0$ for all $T \in A$. We consider the following 3 generalized methods.

1. $y = \{y_i\}_{i \in N}$ belongs to the A–proportional solution of (N, A, t, v) if there exists $\alpha > 0$ such that $y(T) = \alpha v(T)$ for all $T \in A$, $y_i \geq 0$ for all $i \in N$, $\sum_{i \in N} y_i = t$.

2. $y = \{y_i\}_{i \in N}$ belongs to the A–proportional nucleolus of (N, A, t, v) if y belongs to the nucleolus w.r.t. $\{f_T\}_{T \in A}$ with $f_T(z) = z(T) / v(T)$ on the set $\{z = \{z_i\}_{i \in N} : z_i \geq 0, \sum_{i \in N} z_i = t\}$.

3. $y = \{y_i\}_{i \in N}$ belongs to the A–weighted entropy solution of $(N, A, t, v))$ if y minimizes

$$\sum_{T \in A} z(T)(ln(z(T) / v(T)) - 1)$$

on the set $\{z = \{z_i\}_{i \in N} : z_i \geq 0, \sum_{i \in N} z_i = t\}$.

For each A, $t > 0$, v with $v(T) > 0$, the A–proportional nucleolus and the A-weighted entropy solution are nonempty and define uniquely $y(T)$ for each $T \in A$.

The A-proportional solution is nonempty for all $t > 0$, v with $v(T) > 0$ if A is a minimal covering of N.

The A-proportional nucleolus is contained in the A-proportional solution for all $t > 0$, v with $v(T) > 0$ if A is a partition of N.

The A-weighted entropy solution is contained in the A-proportional solution for all $t > 0$, v with $v(T) > 0$ if A is a partition of N.

A vector $y = \{y_i\}_{i \in N}$ belongs to the A-weakly proportional solution of (N, A, t, v) if $y_i \geq 0$ for all $i \in N$, $\sum_{i \in N} y_i = t$, $y(S) / v(S) = y(Q) / v(Q)$ for $S, Q \in A$ with $S \cap Q = \varnothing$.

Necessary and sufficient condition on A that ensures nonemptyness of the A-weakly proportional solution for all $t > 0$, v with $v(T) > 0$ is obtained.

Necessary and sufficient condition on A that ensures nonemptyness of the intersection of the A- proportional nucleolus with the A-weakly proportional solution for all $t > 0$, v with $v(T) > 0$ is obtained.

A vector $y = \{y_i\}_{i \in N}$ is the uniform losses solution of the claim problem (N, t, x), where $x_i \geq 0$, $t > 0$, if there exists $\mu \in R^1$ such that $y_i = (x_i - \mu)_+$, $\sum_{i \in N} y_i = t$, $y_i \geq 0$.

Then y satisfies the following 3 conditions.

C1. $y_i \geq 0$, $\sum_{i \in N} y_i = t$, and if $x_i - y_i > x_j - y_j$, then $y_j = 0$.

C2. $\{y\}$ is the nucleolus w.r.t. $\{f_i\}_{i \in N}$, where $f_i(z) = z_i - x_i$, on the set $\{z = \{z_i\}_{i \in N} : z_i \geq 0, \sum_{i \in N} z_i = t\}$.

C3. y minimizes $\sum_{i \in N} (z_i - x_i)^2$ on the set $\{z = \{z_i\}_{i \in N} : z_i \geq 0, \sum_{i \in N} z_i = t\}$.

Moreover, if y satisfies one of conditions C1,C2,C3, then y is the uniform losses solution of the claim problem (N, t, x). Therefore, for generalized claim problem (N, A, t, v) with $v(T) \geq 0$ we may define 3 generalizations of the uniform losses solution (using conditions C1, C2, C3 respectively): the A-uniform losses solution, the A-nucleolus, and the A-least square solution. We also define the A-weakly uniform losses solution, where we demand that $v(S) - y(S) > v(Q) - y(Q)$ implies $y(Q) = 0$ only for $S \cap Q = \varnothing$.

We obtain conditions on A that ensure the nonemptyness of the A-uniform losses solution, the nonemptyness of the A-weakly uniform losses solution, the nonemptyness of intersection of the A-nucleolus with these solutions. The results are similar to the corresponding results for generalizations of proportional solution.

References

[1] Moulin, H., 2002. Axiomatic cost and surplus sharing. Handbook of Social Choice and Welfare, Volume 1, Chapter 6. Arrow K.J., Sen A.K. and Suzumura K. eds., 289-357

[2] Jo Thomson, W., 2003. Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: a survey. Mathematical Social Sciences 45, 249-297.

About one Collective Risk Model

Maria Nikitina[1] and **Lidia Zolotukhina**[2]

[1]*State Marine Technical University of St. Petersburg,*
International Banking Institute
Russia
maryaniki@gmail.com
[2]*State Marine Technical University of St. Petersburg,*
Russia
lidia.zolotukhina@gmail.com

Keywords: *Collective risk model, Ruin probability, Risk theory, Insurance*

Abstract: *We considered a multivariate generalization of the classical collective risk model, in which each component is dependent on the others. The exact solutions and asymptotic approximations to the probability of ruin were obtained. Some examples are given for graphical illustration.*

One of the most important problems in risk theory is the calculation of the probability of ruin an insurance company. In the classical collective risk model it is assumed that claim sizes are independent identically distributed random variables, namely the claims` flow is independent. However, in reality the insurance company underwrites contracts which are related to different types of claims. For example, let us consider the following types of policies: inhabited house fire insurance and flood insurance. In case of damage by fire the house can be also flooded by fire-fighting operations. So the insurance company will have to compensate twice the same insurance services.

So we considered a multivariate generalization of the classical collective risk model, in which each component is dependent on the others. For example, if we have two types of insurance contracts the surplus of this company is defined by

$$R(t) = u + ct - S^{(1,0)}(t) - S^{(0,1)}(t) - S^{(1,1)}(t)$$

where

u is the initial capital;

c is the insurance premium income per unit of time;

$S^{(1,0)}(t), S^{(0,1)}(t), S^{(1,1)}(t)$ are payment processes of the first, the second or both types of insurance contract.

As a special case of multidimensional collective risk model the two-dimensional model was computer simulated. Changing the correlation index between claim sizes of the first and the second type of insurance contract we defined an influence of dependent claim sizes on probability of ruin. The exact solutions and asymptotic approximations to the probability of ruin were obtained. Some examples are given for graphical illustration.

References

[1] Korolev V., Bening V., Shorgin S., Mathematical Foundations of Risk theory, Fizmatlit, Moscow, 2007
[2] Khokhlov Yu.S., Ivanova N.L., 2007, Multidimensional collective risk model, Journal of Mathematical Sciences № 4, C. 6000-6007.

On Applications of Portfolio Theory to the Problems of Economics and Finance

Oleg Nikonov[1] and **Marina Medvedeva**[2]

Ural State Technical University-UPI;
Institute of Mathematics and Mechanics of Russian Acad. Sci ,
Yekaterinburg, Russia
[1]*olegnikonov@yahoo.com*
[2]*aspr@mail.ustu.ru*

Keywords: *Dynamic portfolio reconstruction, Game-theoretical approach, Non-traditional portfolio theory applications*

In the paper some problems are considered motivated by applied financial and economics issues. The problems under consideration are in some way related to the theory of portfolio investment that goes back to the papers [1-2].

The first section of the paper deals with dynamic problem of financial portfolio management. In contrast to traditional settings [3] we assume that dynamics of the averaged characteristics of financial assets $x_i\ , i = 0,1, ...N,$ are described by differential inclusions

$$\frac{dx}{dt} \in Q(t,x)\ ,\quad x(t_0) = x^0\ ,$$

whereas in the classical approaches stochastic differential equations are usually used for description of prices dynamics. Using approaches of [4-6] we construct a control strategy, which guarantees the efficiency of portfolio $y = (y_0, y_1, \dots , y_N)^T$ in time.

Namely, in the framework of the formalization proposed in the paper that corresponds to the theory of guaranteed control a family of dynamic reconstruction problems of investment portfolio can be considered. In particular, assuming that at the initial moment $t = t_0$ a portfolio y^0 is efficient for the initial state x^0, one can formulate the problem of constructing a strategy $U = U(t,x,y)$ that guarantees the efficiency of investment portfolio and provides the given level of return $\mu[t] \geq \mu[t_0] = \mu^0$ or the given level of risk $\sigma[t] \leq \sigma[t_0] = \sigma^0$.

Here

$$\frac{dy}{dt} \in U(t,x,y)\ ,\ y[t_0] = y^0$$

is an inclusion that describes the dynamic portfolio reconstruction, $\mu[t] = y^T[t] \cdot x[t]$ - dynamic characteristic of return, $\sigma[t]$ - standard deviation of the variable $\mu[t]$ (risk).

If the market is stable, that means that the parameters xi do not change ($Q(t,x) = \{0\}$) then the strategy is trivial U≡0, and there is no need of portfolio reconstruction.

Solvability conditions of the problem allow us to define an index of stability of financial market. Numerical examples related to the present financial crisis are also presented.

In the second part of the paper we show that the portfolio theory initially developed for risky securities (stocks) could be applied to other objects. Among the research of this area indicate [7-8]. In the present paper we consider several situations where such an application is reasonable and seems to be fruitful. Thus, we consider a model that corresponds to a new class of so called synthesized financial instruments, which are the combinations of real financial assets and active strategies of their usage. We also consider the problems of constructing the efficient portfolio of banking services, the portfolios of counteragents of a firm and the portfolios of technologies.

References

[1]. H.Markowitz, Portfolio selection, // J. Finance, 7, 77–91.(1952).

[2] J.Tobin, Liquidity preference as behavior towards risk // Rev. Economic Stud., (1958).

[3] R.C.Merton, Lifetime portfolio selection under uncertainty: The continuous time case, Rev. Econ. Stat., 51, (1969).

[4] A. Kurzhanskii Control and estimation under condition of uncertainty, Moscow: Nauka (1977).

[5] N.N.Krasovskii, A.I.Subbotin, Game-theoretical control problems, Berlin: Springer, (1988).

[6] O.Nikonov, On a Set-Valued Dynamic Model of Investment Portfolio Reconstruction, Modeling and Control of Economic Systems : proc. IFAC Sympos. (2001; Klagenfurt)}, New York etc.:Elsevier, 181–186, (2002).

[7] Helfat,C Investment choices in industry, MT, Cambrige, 1988.

[8] Awerbuch, S. Portfolio-based Electricity Generation Planning: Policy Implications for Renewables and Energy Security // Mitigation and Adaptation Strategies for Global Change, 11(3), 693-710, 2006.

Values for Games on the Cycles of a Digraph

William Olvera[1] and **Francisco Sánchez**[2]

[1,2]*Centro de Investigación en Matemáticas A.C. ,*
Mexico
[1]*e-mail: william@cimat.mx*
[2]*e-mail: sanfco@cimat.mx*

Keywords: *Cooperative games; Digraphs; Game theory; Shapley value.*

There are problems related with walks into digraphs in which it is necessary to quantify the importance of the nodes in order to measure the contribution of each node in the digraph. With this measurement it is possible to make comparative studies among the nodes, ranking them or just determining their importance in a process. In this work, we restrict walks to cycles, motivated by the minimal cost-to-time ratio cycle problem formulation: a tramp steamer is free to choose its ports of call and the order in which it calls on them; the voyage from port i to port j earns an established profit Pij and it needs a time Tij to do it; so, it is necessary to find the ports and the visit order such that the tramp steamer maximizes its cost-to-time ratio. The solution of this problem is well-known and there exists an algorithm for finding the optimal route, a cycle. We propose a method to measure the importance of each port for the tramp steamer based on cooperative games.

Many situations can be modeled as a cooperative game. In our case we adapt that technique and motivated by the problem presented before, we introduce a new cooperative game defined on the cycles of a digraph, where a real number is assigned to each cycle on the digraph. This real number represents the worth that the tramp steamer obtains if it visits all the nodes in the cycle in the sequential order. Since every cycle has an assigned worth, it is possible to use this information and cooperative game techniques to measure how each node contributes to the possible routes of the tramp steamer. We use an axiomatic characterization based on the set of axioms which characterizes the Shapley value (symmetry, linearity, efficiency and nullity) but we adapt them for this new class of cooperative games, obtaining a solution. Also, we show that this solution is related with the Shapley value of a game in characteristic function form.

We also consider the organization of the nodes of a digraph into groups before the negotiation process. In the tramp steamer problem this situation refers to ports grouping in trade unions or political organizations such as countries and we present a characterized solution, where the payments are assigned in a two-step procedure, applying a similar idea of the Owen value.

A third modification of the original model presented before occurs when a fixed real number must be the allocation amount to be distributed among the players. This variation could happen when an external authority or a governor organism determines the amount to be distributed in the set of players. The fixed amount can be greater or lesser than the players can obtain by their negotiation power. So, it is necessary to provide a distribution mechanism of the excess or lack resulting from the external manipulation. We present this situation in context with games on the cycles of a digraph and we show an axiomatic solution based on the idea of an equal allocation of the possible difference between the amount that players can obtain by themselves and the fixed allocation amount.

Fishy Gifts: Bribing with Shame and Guilt

David Ong

Peking University
HSBC School of Business
Shenzhen, 518055, China
dvdong@gmail.com

Keywords: *Psychological contracting, Bribery, Guilt, Shame, Gifts*

The following is a model of psychological contracting with unmonitorable performance, implicit offers, and screening for non-performance by the announcement of the expectation of performance. It is motivated by the $250 billion prescription drug industry, which spends $19 billion per year on marketing to US doctors, mostly on `gifts', and often, as at Yale, with no monitoring for reciprocation. In one revealing incident, a drug firm representative closed her presentation to Yale medical residents by handing out $150 medical reference books and remarking, "one hand washes the other." By the next day, half the books were returned. I model this with a one shot psychological trust game with negative belief preferences and asymmetric information. I show that the `shame' of accepting a possible bribe can screen for reciprocation inducing `guilt'. An announcement can extend the effect. Current policies to deter reciprocation might aid such screening.

Games with Differently Directed Interests and Management Applications

Guennady A.Ougolnitsky

Southern Federal University,
Rostov-on-Don,
Russia,
ougoln@mail.ru

Keywords: *Hierarchical games, Management applications*

Abstract: *In games with differently directed interests payoff functions of the players contain terms which describe their coincident, antagonistic, or both types of interests. Such game theoretic models have many natural applications in management problems.*

A general model of game with differently directed interests may be represented as follows (case n=2 is considered for simplicity):

$$\begin{aligned} u_1(x_1,x_2) &= f(x_1,x_2) + g(x_1,x_2) + f_1(x_1,x_2) \to \max, x_1 \in X_1; \\ u_2(x_1,x_2) &= f(x_1,x_2) - g(x_1,x_2) + f_2(x_1,x_2) \to \max, x_2 \in X_2. \end{aligned} \tag{1}$$

where terms f describe coincident interests, terms g - antagonistic interests, and terms f_1 , f_2 - independent interests of the players. Specific cases of the model (1) are games with partly coincident interests

$$\begin{aligned} u_1(x_1,x_2) &= f(x_1,x_2) + f_1(x_1,x_2) \to \max, x_1 \in X_1; \\ u_2(x_1,x_2) &= f(x_1,x_2) + f_2(x_1,x_2) \to \max, x_2 \in X_2. \end{aligned} \tag{2}$$

and games with partly antagonistic interests

$$\begin{aligned} u_1(x_1,x_2) &= g(x_1,x_2) + f_1(x_1,x_2) \to \max, x_1 \in X_1; \\ u_2(x_1,x_2) &= -g(x_1,x_2) + f_2(x_1,x_2) \to \max, x_2 \in X_2. \end{aligned} \tag{3}$$

For example, a well-known Germeyer-Vatel model [1]

$$\begin{aligned} u_1(x_1,x_2) &= f(a_1 - x_1, a_2 - x_2) + f(x_1) \to \max, x_1 \in X_1; \\ u_2(x_1,x_2) &= f(a_1 - x_1, a_2 - x_2) + f(x_2) \to \max, x_2 \in X_2. \end{aligned} \tag{4}$$

is a specific case of a game with partly coincident interests (2) for which an existence of strong equilibrium is proved.

There are many natural applications of models (1)-(3) to the management problems. Thus, a model of motivation (impulsion) management [2]

$$\begin{aligned} u_1(x_1,x_2) &= x_1 f(x_2) + f_1(x_1,x_2) \to \max, 0 \le x_1 \le 1; \\ u_2(x_1,x_2) &= (1-x_1) f(x_2) + f_2(x_1,x_2) \to \max, 0 \le x_2 \le 1, \end{aligned} \qquad (5)$$

belongs to the type (3). In our previous paper [3] a specific case of the model (2) was considered, namely a pricing model of hierarchical control of the sustainable development of the regional construction works complex

$$u_L(\overline{p}, p_0, p) = \delta p \alpha(p) - M \rho(p, p_0) \to \max$$

$$0 < p_0 \le \overline{p} \le p_{\max}$$

$$u_F(\overline{p}, p_0, p) = p\alpha(p) + p\xi(p)(1-\alpha(p)) \to \max$$

$$0 \le p \le p_{\max}$$

Here p is sales price; p_0 – normative price of social class residential real estate development; $\overline{p}$ - limit price of social class residential real estate development; M >>1– penalty constant; $\rho(p,p_0) = \begin{cases} 0, p \le p_0 \\ 1, p > p_0 \end{cases}$; δ - Administration bonus parameter for social class residential real estate development sales; p_{max} – «overlimit» price of social class residential real estate development (there are no sales if p > p_{max}); $\alpha(p)$ is share of residential real estate development bought by Administration with warranty; $\xi(p)$ - share of another residential real estate development successfully sold by Developer without help.

There are many another examples of game theoretic models of the described type in management, such as models of resource allocation considering private interests, models of organizational monitoring optimization, models of reward systems, corruption models and so on. The games are considered as hierarchical ones.

References

[1] Germeyer Yu.B., Vatel I.A. Games with hierarchical vector of interests, Izvestiya AN SSSR. Technical cybernetics. 1974. Vol.3. (in Russian).

[2] Ougolnitsky G.A. Game theoretic modeling of the hierarchical control of sustainable development , Game Theory and Applications. 2002. Vol.8. N.Y., Nova Sci. Publ.

[3] Ougolnitsky G.A. Optimization and game theoretic modeling of real estate development, Contributions to game theory and management. Vol. III. Collected papers presented on the Third International Conference Game Theory and Management / Editors L.Petrosjan, N.Zenkevich. – SPb.: Graduate School of Management SBbU, 2010.

Mathematical Model of Diffusion in Social Networks

Elena Parilina and **Olga Bogdanova**

Saint-Petersburg State University
Faculty of Applied Mathematics and Control Processes
Russian Federation
calipso.ya@gmail.com
barlena@gmail.com

Keywords*:* *Social network, Diffusion function, Mean-field theory*

At present social networks uniting people either in real or virtual communication are widespread. Most people are the users of the Internet as well as special social Internet projects such as www.facebook.com, www.odnoklassniki.ru, www.livejournal.com, etc. Social networks attract great attention of market analysts whose main task is to find new effective strategies to have a market for goods and services. It is social networks that let spread information quickly from one network user to another, i.e if a network user possesses some product or service, those network users who communicate with him can also have the same things with a high probability. In this way the network users' "infection" takes place.

The work examines the social network consisting of the set of users and the set of connections. Each network user in each time period (time is discrete) can be in one of the two states: active (possessing the product or service) and potential (not possessing the product or service, but able to buy it on condition that some of his active neighbours let him have necessary information about the product or service). The paper examines the dynamic model, in other words, in each time period the network users choose one of the two strategies: to become an active user (an owner of the product or service) or not to become an active user. The main property of the social network is the distribution of connections, i.e. the probability distribution which shows the part of the users with the certain number of the neighbours in the network.

The paper assumes that a network user becomes an active user with some function (diffusion function) which is the function of the neighbours` total number and the number of the active neighbours of this user. Some probability model of the dynamics of the active network users` number is suggested. Using the main dynamics equation the condition of the network equilibrium (stationary state of the network) is defined. The work examines some special types of the diffusion function for which the conditions of the network equilibrium are derived.

The numerical experiments for different types of networks (networks with different distributions of connections) are made.

References

[1] Centola D.’‘Failure in Complex Social Networks'', 2009, Journal of Mathematical Sociology, 33: pp.64 - 68.
[2] López-Pintado D. ‘‘Diffusion in complex social networks'', 2008, Games and Economic Behavior Volume 62, Issue 2, pp. 573-590.
[3] Newman, M. E. J. “The Structure and Function of Complex Networks”, 2003, SIAM Review, 45 (2), pp. 167-256.
[4] Vega-Redondo F. ‘‘Complex Social Networks'', 2007, NY, Cambridge University Press.

The Fixed Point Method versus the KKM Method

Sehie Park

The National Academy of Sciences, Republic of Korea, Seoul 137–044; and Department of Mathematical Sciences, Seoul National University, Seoul 151 - 747, KOREA
E-mail: shpark@math.snu.ac.kr

Keywords: *Abstract convex space, Generalized G- convex space, KKM theorem, KKM space, Partial KKM principle, Fixed point, Nash equilibria*

The KKM theory, first called by the author, is the study on applications of equivalent formulations or generalizations of the KKM theorem due to Knaster, Kuratowski, and Mazurkiewicz in 1929. The KKM theorem provides the foundations for many of the modern essential results in diverse areas of mathematical sciences including game theory.

Let $\langle D\rangle$ denote the set of all nonempty finite subsets of a set D.

An abstract convex space $(E, D; \Gamma)$ consists of a topological space E, a nonempty set D, and a multimap $\Gamma : \langle D\rangle \multimap E$ with nonempty values $\Gamma_A := \Gamma(A)$ for $A \in \langle D\rangle$.

We have the following diagram for triples $(E, D; \Gamma)$:

Simplex $\Rightarrow$ Convex subset of a t.v.s. $\Rightarrow$ Lassonde type convex space

$\Rightarrow$ H-space $\Rightarrow$ G-convex space $\Leftrightarrow$ φ_A-space $\Rightarrow$ KKM space

$\Rightarrow$ Space satisfying the partial KKM principle

$\Rightarrow$ Abstract convex space.

In this paper, we compare the fixed point method and the KKM method in nonlinear analysis. Especially, we consider two methods in the proofs of the following important theorems with the chronological order:

(1) The von Neumann minimax theorem,

(2) The von Neumann intersection lemma,

(3) The Nash equilibrium theorem,

(4) The social equilibrium existence theorem of Debreu,

(5) The Gale-Nikaido-Debreu theorem,

(6) The Fan-Browder fixed point theorem, and

(7) Generalized Fan minimax inequality.

Theorems (1)-(6) are usually obtained by the Brouwer or Kakutani fixed point theorems on convex subsets of topological vector spaces. In this paper, by applying the KKM theory method, Theorems (1)-(3) and (6)-(7) can be generalized to appropriate abstract convex spaces. Moreover, Theorems (4) and (5) can be extended by applying a fixed point theorem for acyclic maps.

We give some of our generalizations of the above theorems:

The Nash equilibrium theorem Let $\{(X_i;\Gamma_i)\}_{i=1}^n$ *be a finite family of compact abstract convex spaces such that* $(X;\Gamma) = (\prod_{i=1}^n X_i;\Gamma)$ *satisfies the partial KKM principle and, for each* i *, let* $f_i, g_i : X = X^i \times X_i \to R$ *be real functions such that*

(0) $f_i(x) \leq g_i(x)$ *for each* $x \in X$ *;*

(1) for each $x^i \in X^i$ *,* $x_i \mapsto g_i[x^i, x_i]$ *is quasiconcave on* X_i *;*

(2) for each $x^i \in X^i$ *,* $x_i \mapsto f_i[x^i, x_i]$ *is u.s.c. on* X_i *; and*

(3) for each $x_i \in X_i$ *,* $x^i \mapsto f_i[x^i, x_i]$ *is l.s.c. on* X^i *.*

Then there exists a point $\widehat{x} \in X$ *such that*

$$g_i(\widehat{x}) \geq \max_{y_i \in X_i} f_i[\widehat{x}^i, y_i] \quad \text{for all } i = 1, 2, \ldots, n.$$

The Gale-Nikaido-Debreu theorem *Let* $(E, F, \langle \cdot,\cdot \rangle)$ *be a dual system of t.v.s.* E *and* F *such that the bilinear form* $\langle \cdot,\cdot \rangle$ *is continuous on compact subsets of* $E \times F$ *. Let K and L be compact convex subsets of t.v.s. E and F, resp., such that K is admissible; and P the convex cone* $\bigcup\{rK \mid r \geq 0\}$ *. Let* $T : K \multimap L$ *be an acyclic map such that* $\langle x, y \rangle \geq 0$ *for all* $x \in K$ *and* $y \in T(x)$ *. Then there exists* $\bar{x} \in K$ *such that* $T(\bar{x}) \cap P^+ \neq \varnothing$ *.*

Here, for a convex cone P of E , the *dual cone* is defined by

$$P^+ := \{y \in F \mid \langle p, y \rangle \geq 0,\ p \in P\}.$$

The Fan-Browder fixed point theorem *An abstract convex space* $(X,D;\Gamma)$ *is a KKM space iff for any maps* $S : D \multimap X, T : X \multimap X$ *satisfying*

(1) $S(z)$ *is open* [*resp., closed*] *for each* $z \in D$ *;*

(2) for each $y \in X, \mathrm{co}_\Gamma S^-(y) \subset T^-(y)$ *; and*

(3) $X = \bigcup_{z\in M} S(z)$ *for some* $M \in \langle D \rangle$ *,*

T *has a fixed point* $x_0 \in X$ *; that is* $x_0 \in T(x_0)$ *.*

Generalized Fan minimax inequality *Let* $(X,D;\Gamma)$ *be an abstract convex space satisfying the partial KKM principle,* $f : D \times X \to \overline{R}, g : X \times X \to \overline{R}$ *extended real functions, and* $\gamma \in \overline{R}$ *such that*

(1) for each $z \in D$ *,* $\{y \in X \mid f(z,y) \le \gamma\}$ *is closed;*

(2) for each $y \in X$ *,* $co_\Gamma\{z \in D \mid (z,y) > \gamma\} \subset \{x \in X \mid g(x,y) > \gamma\}$ *;*

(3) some compactness condition holds for $G(z) := \{y \in X \mid (z,y) \le \gamma\}$ *.*

Then either (i) there exists a $\widehat{x} \in X$ *such that* $f(z,\widehat{x}) \le \gamma$ *for all* $z \in D$ *; or (ii) there exists an* $x_0 \in X$ *such that* $g(x_0,x_0) > \gamma.$

The following references are basic.

References

[1] S. Park, *Elements of the KKM theory on abstract convex spaces*, J. Korean Math. Soc. 45(1) (2008), 1–27.

[2] S. Park, *New foundations of the KKM theory*, J. Nonlinear Convex Anal. 9(3) (2008), 331–350.

[3] S. Park, *Equilibrium existence theorems in KKM spaces*, Nonlinear Analysis 69 (2008), 4352–4364.

On a Proportional Excess Invariant Solution for NTU Games

Sergei Pechersky

St.Petersburg Institute for Economics and Mathematics RAS, and
European University at St.Petersburg;
1 Tchaikowski str., St.Petersburg, RUSSIA 191187
Tel/fax: +7-812-2737953;
specher@emi.nw.ru

Keywords: *Cooperative NTU games, Proportional excess, Proportional excess invariant solution.*

Abstract: *A TU games solution which is invariant with respect to proportional excess is generalized to NTU games. The existence theorem is proved.*

Proportional excess invariant solution (i.e. solution, which is invariant with respect to proportional excess) for positive TU games was defined by E.Yanovskaya (2004) in a following manner. Let N be a set of players. A solution Ψ on the set G_{N+} of all positive TU games with players in N is called proportional excess invariant, or shortly proportional invariant (p.i.-solution in what follows) if for every two games $(\mathrm{N,v})$, $(\mathrm{N,v}) \in \mathrm{G_{N+}}$ and every $x \in X(\mathrm{N,v})$, $y \in X(\mathrm{N,v})$, where $X(\mathrm{N,v})=\{x \in \mathbb{R}_+^{\mathrm{N}} : x(N) = v(N)\}$ the equalities

$$v(S)/x(S) \;=\; w(S)/y(S) \quad \text{for every} \;\; S \subset N$$

imply

$$x = \Psi(N,v) \quad \Leftrightarrow \quad y = \Psi(N,w).$$

One of these solutions, which is characterized by efficiency, anonymity, restricted linearity, positive homogeneity and proportional invariance properties, can be defined by

$$\Psi\ (N,v) \;=\; \max_{x\in X(N,v)} \sum_{s\subset N} c(s)v(S)\ln x(S)$$

for some non-negative numbers $c(s) \neq 0$, $s \leq\ n$ - 1, $n =\mid N\mid$. We put $c(s) = 1$ in what follows. The maximum is attained in a unique positive point, and $x \;=\; \Psi(N,v)$ should be necessarily a solution of the system

$$\sum_{S:i\in S} v(S)\,/\,x(S) = \sum_{S:j\in S} v(S)\,/\,x(S) \text{ for all } i,\ j \in N\,. \qquad (1)$$

Our goal is to generalize this solution to the NTU games, and we will use (1) as a basis for such generalization. Firstly recall the definition of the proportional excess for NTU games. It was defined axiomatically by author (see, for example, Pechersky, 2007).

Let CG_+^N denotes the space of NTU games possessing the following properties for every $S \subset N$, where N is the finite set of players:

(a) $V(S)$ is positively generated, i.e. $V(S) = (V(S) \cap \mathbb{R}_+^S)$ - $\mathbb{R}_+^S$ and $V_+^S = V(S) \cap \mathbb{R}_+^S$ is a compact set, and every ray $L_x = \{\lambda x : \lambda \ge 0\},\ x \ne \boldsymbol{0} = (0,...,0)$ does not intersect the boundary of $V(S)$ more than once;

(b) $\boldsymbol{0}$ is an interior point of the set $V\,\hat{}\,(S) = V(S) \times R^{N\setminus S}$.

(Of course, all $V(S)$ are comprehensive).

Let v be a positive TU game, i. e. $v(S) > 0$ for every S . Then the corresponding NTU game $V \in CG_+^N$ can be defined by

$$V(S) = \{x \in R_+^S : x(S) \le v(S)\} - R_+^S.$$

For $S \subset N$ a set $\mathrm{V(S)} \subset \mathbb{R}^S$ will be called a *game subset*, if it satisfies (a) and (b). The space consisting of all game subsets satisfying (a) and (b) will be denoted by CG_{N+}^S .

Let $V \in CG_+^N$ be an arbitrary game. Then the *proportional excess* $h_S : CG_{N+}^S \times \mathbb{R}_+^N \to \mathbb{R}$ is defined by

$$h_S(V,x) = 1/\gamma(V(S),x^S)\,,$$

where $\gamma(W,\ y) = \inf\{\lambda > 0 : y \in \lambda W\}$ is the gauge (or Minkowski gauge) function.

If $V \in CG_+^N$ corresponds to a positive TU game v, then $h_S(V,\ x) = v(S)/x(S).$

Now let us modify slightly the definition of p.i.-solution adopting it to the NTU case. A solution ψ (not necessarily single-valued) on CG_+^N is called *proportional invariant* if for every two games $V,\ W \in CG_+^N$ and every $x \in \partial V_+(N),\ y \in \partial W_+(N)$, the equalities

$$h_S(V,x) = h_S(W,y) \text{ for every } S \subset N$$

imply

$$x \in \psi(N,V) \Leftrightarrow y \in \psi(N,W).$$

Theorem 1. There is a p.i.-solution on CG_+^N. For every game $V \in CG_+^N$ it can be defined as the solution of the system

$$x \in \partial V_+(N),$$

$$\sum_{S: i\in S} h_S(V,x) \ = \ \sum_{S: j\in S} h_S(V,x) \text{ for all } i,\ j \in N.$$

The proof of the theorem uses the Brower fixed point theorem. We denote this solution by ψ.

Proposition 1. Let *BG* be a family of non-leveled bargaining games in CG_{N+}. Then there is a unique p.i.-solution on *BG*, and it is the *status quo*-proportional solution.

(See the definition and the properties of the *status quo*-proportional solution in Pechersky 2007, 2008).

Proposition 2. If CG_{N+} corresponds to a positive TU game v, then $\psi(V) = \Psi(v)$.

Proposition 3. P.I.-solution ψ possesses efficiency, anonymity and positive homogeneity properties.

Let us introduce an operation on CG_{N+}. We call it *directional addition*, and define it as follows. Let $A,\ B \in CG_{N+}^S$. Then for every $x \in \mathbb{R}_+^S$ there are exactly two points $y \in \partial A$ and $z \in \partial B$ such that $y = \lambda_x x$ and $z = \mu_x x$ for some positive numbers λ_x and μ_x. Then the *directional sum* of A and B, denoted by $A \oplus_d B$, is defined as follows:

$$A \oplus_d B = \text{comp}\{\cup_x (\lambda_x + \mu_x)x\}, \tag{2}$$

where comp F denotes the comprehensive hull of a set F.

Let now $V,W \in CG_{N+}$ with $V(N) = W(N)$. Define the game $V \oplus_d W$ as follows:

$$(V \oplus_d W)(N) = V(N) = W(N), \tag{3}$$

$$(V \oplus_d W)(S) = V(S) \oplus_d W(S) \text{ for every } S \subset N. \tag{4}$$

Clearly $V \oplus_d W \in CG_{N+}$.

Remark 1. To omit restriction $V(N) = W(N)$ it is possible to use $\frac{1}{2}(\lambda_x + \mu_x)$ in (2) instead of $(\lambda_x + \mu_x)$.

It is not difficult to prove that $h_S(V \oplus_d W, x) = h_S(V, x) + h_S(W, x)$ for every $S \neq N$.

Proposition 4. Let V, $W \in CG_{N+}$ with $V(N) = W(N)$, and $x \in \psi(V) \cap \psi(W)$. Then $x \in \psi(V \oplus_d W)$.

Remark 2. When using the definition mentioned in Remark 1, condition $V(N) = W(N)$ can be omitted. It is also clear that the directional sum $aV \oplus_d (1-a)W$ for every positive number $a < 1$ can be defined. In that case a simple modification of Proposition 4 generalizes the restricted linearity property for TU case.

On A Multistage Link Formation Game

Leon Petrosyan[1] and **Artem Sedakov**[2]

[1,2]*St Petersburg University*
Russia
[1]*spbuoasis7@peterlink.ru*
[2]*a.sedakov@yahoo.com*

Keywords: *Graph, Link formation, Myerson value, Subgame perfect equilibrium*

In the paper we propose an approach of multistage link formation between players. Let (N,v) be a TU game, where $N=\{1,\ldots,n\}$ is a finite set of players and $v:2^N\to\mathbb{R}$ with $v(\varnothing)=0$. Denote by $g^N=\{(i,j),i\in N,j\in N,i\neq j\}$ a complete graph on the player set N where a pair $(i,j)=(j,i),i\in N,j\in N$ is an unordered link connecting i and j.

Let $g\subseteq g^N$. Following Myerson (1977) for some $S\subseteq N$ define S/g as a partition of S into subsets of players that are connected in S by g:

$$S\,/\,g=\{i:i \text{ and } j \text{ are connected in } S \text{ by } g;\, j\in S\},$$

and a characteristic function $v^g(S)=\sum_{T\in S/g}v(T)$. We say that players $i,j\in S$ are connected in S if there is a path from i to j such that all intermediate vertices belong to S.

For any $g\in g^N$ as a solution concept of the TU game (N,v) we use the Myerson value (the Shapley value of the associated TU game (N,v^g)).

Consider an approach of link formation. Suppose that a TU game (N,v) and a graph $g\subseteq g^N$ is given. Let for each $i\in N$ vectors $c_i=\{c_{i1},\ldots,c_{in}\}\in\mathbb{R}^{|N|}$ with $c_{ii}=0$ and $d_i=\{d_{i1},\ldots,d_{in}\}\in\mathbb{R}^{|N|}$ with $d_{ii}=0$ are also given. The component c_{ij} represents a maximal amount that player i is ready to pay player j to form a link (i,j). The component d_{ij} represents a minimal amount on which player i is agreed to be paid by player j to form a link (i,j). Without loss of generality we assume that the link formation starts from the empty graph $g_0 = \varnothing$. Let the players order $\{i_1, \ldots, i_n\}$ is a priori given. We begin from i_1. The player i_1 has exactly n alternatives:

• not to take any action, and the game process goes to the player i_2;

• propose to the player k, $k \neq i_1$ an amount c_{i_1k} to form a link (i_1, k). If $c_{i_1k} \geq d_{ki_1}$ the player k is supposed to accept the proposition, and the game process goes to player i_2.

So the decision process goes to the player i_2. Note that the graph g_0 is changed depending upon the player i_1 choice, and we get a graph g_1. Consider an intermediate stage of link formation. Suppose we have a current graph g_{t-1} and the player i_t who has exactly n alternatives:

• not to take any action, and the game process goes to the player i_{t+1};

• propose to the player k, $k \neq i_t$ an amount c_{i_tk} to form a link (i_t, k) if $(i_t, k) \notin g_{t-1}$. If $c_{i_tk} \geq d_{ki_t}$ the player k is supposed to accept the proposition, and the game process goes to player i_{t+1};

• break the link (i_t, k) if $(i_t, k) \in g_{t-1}$ with player k, $k \neq i_t$ paying to him an amount d_{ki_t}, and the game process goes to player i_{t+1}.

After n stages the link formation process is finished. Finally we get an n person game in extensive form on a graph tree K, each vertex of it represents a current link relationship between players. In terminal vertices on K we obtain players payoffs as Myerson values. Using Kuhn's technique we give an optimal link structure in terms of subgame perfect equilibrium. This result is illustrated by numerical example.

References

[1] Nash J. (1951) Non-cooperative games. Ann. of Math. 54:286–295.

[2] Kuhn H. W. (1953) Extensive Games and Problem Information. Ann. Math Studies 28:193–216.

[3] Myerson R. (1977) Graphs and cooperation in games. Mathematics of Operations Research 2:225–229.

[4] Dutta B., van den Nouweland A., Tijs S. (1998) Link formation in cooperative situations. International Journal of Game Theory 27:245–256.

[5] Petrosjan L., Kusyutin D. (2000) Games in extensive form: optimality and stability, St. Petersburg Univ. Press, St.Petersburg.

[6] Petrosyan L., Sedakov A. (2009) Multistage networking games with full information. Upravlenie bolshimi sistemami 26.1:121–138 (in russian).

Location Games on Graphs

Erich Prisner

Franklin College,
Switzerland
eprisner@fc.edu

Keywords: *Graph, Location game, Pure Nash equilibria*

We consider 2-player location games played on graphs, where the vertices and the edges of the graph may have weights. The graph could be considered to be a physical street network with villages as vertices, the vertex weights being the number of people, possible customers for the players, living there, and the edge weights being the lengths of the corrisponding roads. Each player places a fixed number of locations on some vertices. People may or may not go to some location (to shop), depending on the distance to the closest location, but if they do, they will always choose the closest location, no matter to which one of the 2 players it belongs. The payoff for each player is the total number of customers in his or her locations.

We consider several variants, depending on

• the underlying vertex-weighted and edge-weighted graph G,

• the number k of locations each player can place,

• whether the game is simultanous, or sequential with 2 rounds, or sequential with 2k rounds,

• A (non-increasing) function f describing the tolerance of possible customers to distance. Only f(x) of the customers will go to a location if the closest location has a distance of x.

• The rule how ties—one person having several locations with closest distance to him or her—are broken. Does that person visit any of the locations randomly, or use a 50%-50% split between locations of the two players in that case?

Note that the simultaneous games are symmetric. The games are ususally not zero-sum, except in the case of $f(x) = 1$, where all customers always visit one of the locations.

Note also that there are questions of how well the resulting pattern of locations serves the population as a whole. Home many people visit some of the locations? What is the average length to the closest location for those who do? What is the average length to the closest location over the whole population?

Location games have been initiated by H. Hotelling in 1929 [1], considering the case of location on a straight line. Later, location games for other geometrical patterns have been investigated. Location games on graphs have been investigated by V. Knoblauch in two papers in 1991 and 1995. However, her models considered differ from ours. In [2], location everywhere on the edges is allowed. In [2], location is allowed only on some specified vertices of the graph, and only functions f of the form $f(x) = 1$ for $x \leq m$, and $f(x) = 0$ otherwise, are considered. In this second model, Knoblauch could show that every simultaneous 2-person symmetric game is equivalent to such a simultaneous location game on a graph.

In this talk we are going to discuss the following topics:

• We will discuss the question of what kind of games result from simultaneous location games with a function $f(x) = 1$ (meaning that every person will go to the closest location). We will also discuss the existence and number of pure Nash equilibria of these games.

• We will have a closer look into the case of simultaneous games with a function $f(x) = 1$ on special graphs, like trees or chordal graphs.

• We will discuss the relation to so-called ”'potential games"' [4] for simlu-taneous location games with one location to place for each player. We will also discuss the existence of pure Nash equilibria.

• Some examples of simultaneous location games without pure Nash equilibria will be presented.

• We will discuss the welfare for the public of the resulting locations in some cases, using the measures discussed above.

• We will also discuss some of the sequential versions.

References

[1] H. Hotelling, Stability in competition, Economic Journal 39 (1929) 41-57.
[2] Vicki Knoblauch, Generalizing Location Games to a Graph, Journal of Industrial Economics 34 (1991).
[3] V Knoblauch, Geometric versions of finite games: Prisoner's dilemma, entry deterrence and a cyclical majority parado, International Journal of Game Theory 24 (1995) 165-177.
[4] D. Monderer, L.S. Shapely, Potential Games, in: Games and Economic Behavior, 1996, 124-143.

Incentive Cooperative Condition in Discrete-time Bioresource Management Problems

Anna N. Rettieva

Institute of Applied Mathematical Research Karelian Research Center of RAS
Russia
annaret@krc.karelia.ru.

Keywords: *Bioresource management problem, Time-consistency, Distribution procedure, Incentive cooperative condition, Irrational-behavior-proofness*

Abstract: *The cooperation plays important role in the bioresource management problems, because it causes minimal damage to environment. There are several methodological schemes to maintain the cooperation. Here we consider time-consistent imputation distribution procedure and special conditions for maintenance the cooperation*.

Model with logarithmic payoffs

The fish population grows according to the biological rule

$$x_{t+1} = (\varepsilon x_t)^{\alpha},\ 0 < \alpha < 1\,. \tag{1}$$

We suppose that the utility function of country i is logarithmic. Then the players' net revenue over infinite time horizon are:

$$J_1 = \sum_{t=0}^{\infty} \delta^t \ln(u_t^1),\ J_2 = \sum_{t=0}^{\infty} \delta^t \ln(u_t^2)\,, \tag{2}$$

where $u_t^1, u_t^2 \geq 0$ – countries' catch at time t, $0 < \delta < 1$ – the common discount factor for countries.

For this model we find Shapley value and time-consistent imputation distribution procedure. We consider two conditions which help to maintain the cooperation.

Definition 1. The distribution (ξ_1, ξ_2) satisfies Yeung's condition [3] if

$$\sum_{\tau=0}^{t-1} \delta^{\tau} \beta_i(\tau) + \delta^t V_i(t) \geq V_i(0),$$

for all $t \geq 1$, where $\beta(t) = (\beta_1(t), \beta_2(t))$ – time-consistent imputation distribution procedure.

If this condition is satisfied player i is irrational-behavior-proof, because irrational actions leading to the breaking the cooperative agreement will not bring his payoff below his initial noncooperative payoff.
This condition in our case is equal

$$\xi_i(0) - \xi_i(t)\delta^t \geq V_i(0) - \delta^t V_i(t), i = 1,2. \tag{3}$$

Definition 2. The distribution (ξ_1, ξ_2) satisfies incentive cooperative condition if

$$\beta_i(t) + \delta V_i(t+1) \geq V_i(t), i = 1,2,$$

for all $t \geq 0$, where $\beta(t) = (\beta_1(t), \beta_2(t))$ – time-consistent imputation distribution procedure.

This condition offers an incentive to player i to keep cooperation, because at every step he gets more gain from cooperation than from noncooperative behavior.This condition in our case is equal

$$\xi_i(t) - \delta\xi_i(t+1) \geq V_i(t) - \delta V_i(t+1), i = 1,2. \tag{4}$$

As you can notice that incentive cooperative condition yields Yeung's condition. We proved that Yeung's and cooperative incentive conditions are satisfied for this model.

Model with quadratic payoffs

The fish population grows according to the biological rule

$$x_{t+1} = \varepsilon x_t - u_t^1 - u_t^2, \varepsilon > 1, \tag{5}$$

where $u_t^1, u_t^2 \geq 0$ – countries' fishing efforts at time t.

The players' net revenue over infinite time horizon are:

$$J_1 = \sum_{t=0}^{\infty} \delta^t (pu_t^1 - c(u_t^1)^2), J_2 = \sum_{t=0}^{\infty} \delta^t (pu_t^2 - c(u_t^2)^2), \tag{6}$$

where $0 < \delta < 1$ – the common discount factor for countries, $p > 0$ – price for fish, $c > 0$ – catching cost.

For this model we also find Shapley value and time-consistent imputation distribution procedure. We find conditions on x_0 that guarantee the observance of Yeung's and incentive cooperative condition.

Acknowledgement

The research was supported by the Russian Fund for Basic Research, project 10-01-00089-a.

References

[1] Mazalov V.V., Rettieva A.N. Incentive equilibrium in discrete-time bioresource sharing model, Doklady Mathematics. 2008. V. 423. N. 3. p. 320–322.
[2] Petrosjan L., Zaccour G. Time-consistent Shapley value allocation of pollution cost reduction , J. of Economic Dynamic and Control. 2003. V. 7. p. 381–398.
[3] Yeung D.W.K. An irrational-behavior-proof condition in cooperative differential games , Game Theory Review. 2006. V. 8. N 4. p. 739–744.

ВЕСТНИК САНКТ–ПЕТЕРБУРГСКОГО УНИВЕРСИТЕТА

Серия 8
МЕНЕДЖМЕНТ

Ответственный редактор серии:
Ю.Е. Благов

Издательство Высшей школы менеджмента Санкт-Петербургского университета

Coherent Modeling of Risk in Optimization under Uncertainty

Ralph Tyrrell Rockafellar

University of Washington, Seattle,
Department of Mathematics
Seattle, WA 98195-4350
Fax: (206) 543-0397
rtr@math.washington.edu
and
University of Florida, Gainesville
www.ise.ufl.edu/rockafellar

In many situations in which some kind of optimization is desirable, the decisions that must be made in the present are unable to produce unambiguous future outcomes. They can only affect the probability distributions of various future "costs" or losses, and then even the framework for optimization is called into question.

To get around the difficulty, preferences toward risk must be brought in, and these can involve a lot more ideas than expected utility. Moreover some traditional approaches with safety margins and probabilistic constraints are open to serious criticism. The systematic theory of risk that has been developed in recent years, out of motivations in finance and engineering, offers a clear view of how optimization can be carried out "coherently" despite such issues.

Uncertainty Aversion and Equilibrium in Extensive Form Games

Jörn Rothe

London School of Economics
United Kingdom
J.D.Rothe@lse.ac.uk

Keywords: *Extensive game, Uncertainty aversion, Equilibrium, Choquet expected utility, Rationality*

This paper formulates a rationality concept for extensive games in which deviations from rational play are interpreted as evidence of irrationality. Instead of confirming some prior belief about the nature of non-rational play, we assume that such a deviation leads to genuine uncertainty. Assuming complete ignorance about the nature of non-rational play and extreme uncertainty aversion of the rational players, we formulate an equilibrium concept on the basis of Choquet expected utility theory. We relate the equilibrium concept to the foundations of game theory and discuss the centipede game and the finitely repeated prisoners' dilemma.

According to the principle of sequential rationality, a rational player of an extensive form game regards his opponents as rational even after a deviation from rational play. The internal consistency of this principle is subject of much debate (see, e.g., Aumann (1995, 1996, 1998), Binmore (1996), Reny (1993)). Attributing non-rational deviations to 'trembles' of otherwise perfectly rational players (Selten 1975) is logically consistent, but raises the second concern with the principle of sequential rationality, that is its empirical plausibility. Quite independently of the question whether there exists a rationality concept that implies, or is at least consistent with sequential rationality, the question arises if there is room for an alternative rationality concept, in which deviations from the solution concept are interpreted as evidence of non-rationality. In this paper, we attempt to formulate such a rationality concept on the basis of Choquet expected utility theory.

In a seminal series of papers, Kreps, Milgrom, Roberts, and Wilson (1982) (henceforth KMRW) developed the methodology for analysing games with possibly non-rational opponents. In their models, there is some a priori uncertainty about the rationality of the opponent. Under subjective expected utility, players act as if they possess a probability distribution over the 'type' of the opponents' non-rationality. They maximise utility given their beliefs, and in sequential equilibrium their beliefs are consistent with the play of rational opponents. KMRW have shown how even small degrees of uncertainty about rationality can have large equilibrium effects. They showed that this can explain both intuitive strategic phenomena, particularly in industrial organization, and, at least to some degree, experimental evidence.

One problem in this approach, however, is for an outside observer to specify the probability distribution over the types of the non-rational opponents before experimental or field data are available. A second problem is that analysing the strategic interaction as a game with incomplete information implies that the other players, whether rational or not, can be modelled as 'types', who possess a consistent infinite hierarchy of beliefs about the strategic interaction. Thus, the players in this methodology are not really non-rational; rather, they are rational but have preferences that differ from those that the game attributes to 'rational' players.

In this paper, we argue that a consistency argument addresses both of these problems. A game-theoretic solution concept that singles out rational strategies implicitly defines all other strategies as non-rational. Thus, consistency requires that beliefs about non-rational players should not exclude any of these non-rational strategies. In other words, if the rationality concept is point-valued, the beliefs about non-rational play should include all deviations, and thus must be set-valued. So in this sense, the rationality concept itself pins down beliefs about non-rational play, but excludes subjective expected utility theory (henceforth SEU) as the adequate model of these beliefs. Thus, SEU is not an appropriate framework for beliefs about non-rationality when rationality is endogenous. Thus, this paper argues that, after an opponent deviates from rational play, a rational player faces genuine uncertainty. What matters, then, is the rational player's attitude towards uncertainty. This paper formulates the equilibrium concept for the case in which rational players are completely uncertainty averse. It is this case that has led to the development of decision theories with set-valued and non-additive beliefs as an explanation of the Ellsberg (1961) paradox. Consequently, we base the equilibrium concept on Choquet expected utility theory (henceforth CEU) developed by Schmeidler (1989).

This paper joins a growing literature that applies CEU to games. The first of these were Dow & Werlang (1994) and Klibanoff (1993). Dow & Werlang (1994) consider normal form games in which players are CEU maximisers. Klibanoff (1993) similarly considers normal form games in which players follow maxmin-expected utility theory (Gilboa & Schmeidler 1989), which is closely related to CEU. In Hendon, Jacobsen, Sloth & Tranæs (1995) players have belief functions, which amounts to a special case of CEU. Extensions and refinements have been proposed by Eichberger & Kelsey (1994), Lo (1995b), Marinacci (1994) and Ryan (1997). Epstein (1997a) analysed rationalizability in normal form games. These authors consider normal form games and do not distinguish between rational and non-rational players. The paper closest to ours is Mukerji (1994), who considers normal form games only but argues that the distinction between rational and non-rational players is necessary to reconcile CEU with the equilibrium concept. For normal form games our concepts differ only in motivation and technical team. The present paper mainly concerns extensive form games. Extensive games have been studied by Lo (1995a) and Eichberger & Kelsey (1995). Lo (1995a) extends Klibanoff's approach to extensive games, Eichberger & Kelsey (1995) are the first to use the Dempster-Shafer updating rule (see section 3) in extensive games. They do not distinguish between rational and non-rational players.

This paper is organized as follows: The next section discusses an example. Section 3 presents Choquet expected utility theory and discusses the problem of updating non-additive beliefs. In section 4 we formulate the equilibrium concept for two player games with perfect information. In section 5 we discuss the centipede game and the finitely repeated prisoners' dilemma in order to relate the equilibrium concept to the foundations of game theory. Section 6 elaborates on the extension of the solution concept to general extensive games. Section 7 concludes. There is one appendix on teams of updating non-additive beliefs.

Nash Equilibrium in Games with Ordered Outcomes

Victor Rozen

Saratov State University,
Saratov, Russia
RozenVV@info.sgu.ru

Keywords: *Game with ordered outcomes, An equilibrium point, Nash equilibrium, Mixed strategies, Balanced matrix*

Abstract: *We consider game G with ordered outcomes of two players. In general case, such a game even need not have an equilibrium point when this game is finite. In our previous paper it is shown that for finite game G there exists an equilibrium point in its mixed extension but the existence of Nash equilibrium need not be satisfied. In this report, we find some conditions under which a game G have Nash equilibrium point in mixed strategies. The main result is connected with a concept of balanced matrix.*

We consider games with ordered outcomes of two players in the

$$G = < I, J, A, \omega_1, \omega_2, F >,$$

where I is a set of strategies of player 1, J is a set of strategies of player 2, A is a set of outcomes, ω_1 and ω_2 are orderings on A which represent preferences of players 1 and 2, respectively, $F : I \times J \to A$ is a realization function. In general case, such a game need not have an equilibrium point even when this game is finite.

In our previous paper it is shown that for finite game G there exists an equilibrium point in its mixed extension but the existence of Nash equilibrium need not be satisfied. In this report, we find some conditions under which a game G have Nash equilibrium point in mixed strategies. The main result is connected with a concept of balanced matrix.

Let A be an arbitrary set and $F : I \times J \to A$ be a function. For any vectors $x = (x_i)_{i \in I}$ and $y = (y_j)_{j \in J}$, where $x_i \geq 0, \sum_{i \in I} x_i = 1$ and $y_j \geq 0, \sum_{j \in J} y_j = 1$ put

$$F_{(x,y)}(a) = \sum_{F(i,j)=a} x_i y_j$$

Definition. A matrix $(F(i,j))$ is called balanced if there exist vectors x, y with strict positive components such that for any $i \in I, j \in J$, $a \in A$ the following conditions hold

$$F_{(i,y)}(a) = F_{(x,y)}(a) \text{ and } F_{(x,j)}(a) = F_{(x,y)}(a).$$

Given a game G with ordered outcomes, we can construct its mixed extension (see our previous paper in GTM 2009). A necessary condition for Nash equilibrium point in mixed extension of game G is given by the following theorem.

Theorem 1. Let (x^0, y^0) be Nash equilibrium in mixed extension of game G. Then the matrix which is a restriction of matrix of outcomes $(F(i,j))$ under the pair of spectrums (Spx^0, Spy^0) is balanced.

Corollary 1. Suppose there exists a Nash equilibrium point in mixed extension of game G. Then the matrix of outcomes of game G contains a balanced submatrix.

Note that the last condition is defined only by realization function of game G and not depend on order relations $\omega_i, i = 1,2$.

We now state necessary and sufficient condition for Nash equilibrium in mixed extension of game G. We denote by ϖ_i the extension of order relation ω_i and by $F_{(x,y)}$ the extension of realization function F on the set of probability measures.

Theorem 2. A situation in mixed strategies (x^0, y^0) is Nash equilibrium point in mixed extension of game G if and only if the following conditions hold:

The restriction of matrix of outcomes under the pair of spectrums (Spx^0, Spy^0) is balanced matrix;

$$F_{(i,y^0)} \leq^{\varpi_1} F_{(x^0,y^0)} \text{ for all } i \notin Spx^0;$$

$$F_{(x^0,j)} \leq^{\varpi_2} F_{(x^0,y^0)} \text{ for all } j \notin Spy^0.$$

Corollary 2. A game G have a quite Nash equilibrium point in mixed strategies if and only if its matrix of outcomes is balanced.

Big Mama and the Convergence of Choices

Michel Rudnianski[1] and **Giovanni Sartor**[2]

[1]*IIM CNAM,*
France
michel.rudnianski@cnam.fr
[2]*Law Department, EUI,*
Italy
giovanni.sartor@libero.it

Keywords: *Behaviours, Norms, Coalitions*

I Behaviours and norms convergence.

Analysis of competition and cooperation with the aim of understanding how behaviours may converge and lead to the development of norms, has been the subject of protracted research in social sciences, and particularly in anthropology (Mead [7]). In philosophy and sociology the idea of a convention has been developed, to explain the emergence and persistence of the practice of rules (starting with the seminal work of Lewis [5]). In legal theory a broad discussion has taken place where some authors have argued that the law is essentially a matter of convention (on judges converging on fundamental rules (Hart [3]), while others have denied that conventions explain judicial behaviour. According to Dworkin ([2]), such choices are explained by the attempt to make the law "as good as it can be", in the framework defined with respect to popular will (fairness) and fitness with legal history. On its side Game Theory has provided core concepts and tools, through the development of N-player games (Rapoport [11]), cooperative negotiation concepts (Nash [8]), Kalai & Smorodinsky [4]) or bargaining models (Osborne & Rubinstein [9]), and evolutionary games (Maynard Smith [6]). Thus, N-player games through analyzing the formation of coalitions, have specified the conditions under which these coalitions could lead to qualified majorities, and hence to the adoption of laws and regulations . Likewise, distributive and integrative negotiations have been differentiated (Raiffa [10]), while evolutionary games have been used to analyze the convergence of behaviours (Rudnianski, Ellison & Huo [12]).

Now if reaching an agreement somehow means that the different parties' points of view converge, the development of norms is certainly not a pre-requisite to the occurrence of an agreement, and vice versa an explicit agreement is not a necessary condition for behaviours to converge. Likewise, the occurrence or the non-occurrence of an agreement is most of the time taken into account implicitly in the assessment of the states of the world resulting from implementation of the players' strategies. Nevertheless in some types of social interaction the parties involved may take into account the occurrence of a convergence when assessing a state of the world. This must have been certainly the case for parties involved in the Copenhagen negotiations on climate change: One can reasonably assume that a vast majority of these parties at least would have seen favorably the occurrence of an agreement. Such could be also the case for general assemblies of private companies shareholders, or in the field of legal affairs, for law makers, judges applying imprecise legal corpuses, and thus contributing to develop jurisprudence, or lawyers (Sartor, Rudnianski & all [13]).

The proposed paper will precisely analyze the relations between the occurrence of convergence and the development of norms in three steps

II Big Mama

The first step introduces and analyzes the following metaphor. Alice, Bob and Carol just went out of a ballet, and they want to have supper. All restaurants nearby are already closed, except "Big Mama" which takes its name after the woman who is both the waitress and the cook of the place. Big Mama agrees to serve them supper but warns: "you can choose from the menu the course you want, but it is already late and I am alone in the kitchen. Hence the more different courses you ask, the more you will have to wait, and the courses that are more wanted will be served before the others"

If the three parties are rational, their decisions will obviously depend on the utilities they associate with each course, and that are based both on their personal tastes and the waiting time

On the basis of those utilities, the paper analyzes:

- the conditions under which Alice, Bob and Carol will have their supper served by Big Mama
- how coalitions between the parties can form
- the conditions under which all parties may converge and elect the same course.

III Generalization

The second part of the paper extends the analysis in different directions associated respectively with:

- the decision rules
- the number of players
- the total number of courses figuring on the menu
- the number of different courses allowed by Big Mama
- the discount factor deriving from the time required to be served.

For some particular sets of the above variables, the solutions found are interpreted in the light of the Condorcet Paradox.

Now, it is a well known fact that the repetition of a game may sometimes facilitate either cooperation between the players, as is the case for instance with the iterated Prisoners Dilemma (Axelrod [1]), or the occurrence of an agreement (as shown by Rubinstein's bargaining model [9]). In that perspective the paper will analyze how repetition of the game may reshape the coalitions, and the conditions under which an agreement can occur, and the related norms develop.

IV Application to the development of jurisprudence

The third and last part of the paper will apply the findings of the Big Mama metaphor's to the case of judges who may differently understand the legal texts while believing nevertheless that convergence toward a single interpretation of the law (i.e. toward jurisprudence) is a matter of importance.

The paper will in particular analyze the case of compensatory damages, assuming that a legal article stipulates that "the one who causes a damage has to compensate it", and considering then the following three possible interpretations of the term "damage":

1) damage means only financial damage

2) damage also includes damage to health even if there is no financial impact(for instance, the damage caused is a reduced capacity to practice sport)

3) on the top o the two previous damages, moral damage has also to be taken into account.

It will be assumed that the judges only focus on the social value that can derive from Justice, and are not considering the personal advantages they could obtain by adopting a certain behavior.

References

[1] Axelrod Robert, The Evolution of Cooperation Basic Books, New York, 1984
[2] Dworkin Ronald M., Law's Empire, Kermode, London, 1986.
[3] Hart H. L. A, Postscript, in The Concept of Law, P. Bulloch and J. Raz, eds., Oxford University Press, Oxford, 2nd ed., 1994, pp. 238–76.
[4] Kalai, E. and Smorodinsky M, Other Solutions to the Nash's Bargaining Problem, Econometrica 43, 513-18, 1975
[5] Lewis David K., Conventions: A Philosophical Study, Harvard University Press, Cambridge, Mass., 1969.
[6] Maynard Smith John, Evolution and the Theory of Games, Cambridge University Press, 1982
[7] Mead Margaret., Cooperation and Competition Among Primitive Peoples, Mc Graw Hill, New York, 1937
[8] Nash John E., The Bargaining Problem, Econometric 18, p 155-162, 1950
[9] Osborne Martin J.., Rubinstein Ariel, Bargaining and Markets, Academic Press, San-Diego, 199
[10] Raiffa Howard, Negotiation Analysis: the Science and Art of Collaborative Decision Making, Belknap Press of Harvard University Press, Cambridge MA, 2002
[11] Rappoport Anatol, N-Person Game Theory, Dover Publications, New-York, 1970
[12] Rudnianski Michel, Huo Su, Ellison David, Compliance Pervasion and the Evolution of norms: The Game of Deterrence Approach, Proceedings of the GTM 2008 conference, pp 383-414, St Petersburg, 2008
[13] Sartor Giovanni, Rudnianski Michel Rotolo Antonino, Riveret Régis, Mayor Eunate, Why lawyers are nice (or nasty): a game-theoretical argumentation exercise, Proceedings of the ECAIL Conference, Barcelona 2009

МАТЕМАТИЧЕСКАЯ ТЕОРИЯ ИГР И ЕЕ ПРИЛОЖЕНИЯ

Выходит ежеквартально с января 2009 года

Game for New Insights for Leadership Assessment and Development

Chaudhary Imran Sarwar

CEO-Creative Researcher, 20, A-I,
Township, Lahore, Pakistan

Keywords: *Leadership development*, *Intrinsic desire for leadership development, Inherent potential for leadership development*, *Antecedents of leadership*

This research focuses on game for new insights for leadership assessment and development. It introduces theory on triads of typical-maximal-ideal (a) individually considerating, (b) intellectually stimulating, (c) leading by exception (active), (d) idealized influence (attributed), (e) idealized influence (behavioral), and (f) contingently rewarding leadership performances (for example via triad of typical, maximal, and ideal contingently rewarding leadership performances) adding diversification and precision to leadership assessment. It explores the proposition that within each triad - each of typical, maximal, and ideal leadership performances is theoretically and conceptually distinct and supports this distinction through databased empirical analyses by using mean difference via one sample t-test and one way analysis of variance. Thereafter, it uses each triad of the distinct typical, maximal, and ideal leadership performances to introduce and empirically test the mechanism to quantify respondents' intrinsic desire and inherent potential to enhance their respective leadership performances. Finally, it suggests precedents of each leadership performance and presents implications for leadership development training on the basis of correlations and multiple regression analyses.

Coalition Homomorphisms of Games with Preference Relations

Tatiana Savina

Russian Federation
suri-cat@yandex.ru

Keywords: *Game with preference relations, Coalition homomorphism, Homomorphism, Acceptable outcomes*

Abstract: *Let G be a game with preference relations of players I. Then we can construct a game of coalition if strategies and preference relations of coalitions are given. Let G, H be two games with preference relations. A homomorphism from game G into game H is said to be a coalition homomorphism if it preserves preference relations for coalitions. We study the following problems. We find conditions under which a homomorphism of games is a homomorphism for games of coalitions. We stand a correspondence between acceptable outcomes of games which are in homomorphic relations of various types. We find connections between acceptable outcomes of game with preference relations and a-core for games with payoff functions.*

1. The cooperative aspect of a game is connected with its coalitions. Consider a game of players $N = \{1,\ldots,n\}$ with preference relations

$$G = \langle (X_i)_{i\in N}, A, (\rho_i)_{i\in N}, F\rangle.$$

For any coalition $T \subseteq N$ we need define its set of strategies X_T and its preference relation ρ_T. We consider X_T in the form

$$X_T = \prod_{i\in T} X_i$$

and a preference relation for coalition T in one of the following ways:

A) $a_1 \overset{\rho_T}{\lesssim} a_2 \Rightarrow (\forall i \in T)\, a_1 \overset{\rho_i}{\lesssim} a_2$ – Pareto concordance of preferences of players,

OR

B) $\begin{cases} a_1 \overset{\rho_T}{<} a_2 & \Rightarrow \quad (\forall i \in T)\, a_1 \overset{\rho_i}{<} a_2, \\ a_1 \overset{\rho_T}{\sim} a_2 & \Rightarrow \quad (\forall i \in T)\, a_1 \overset{\rho_i}{\sim} a_2 \end{cases}$ – modified Pareto concordance of preferences of players.

2. In this paper we consider the following optimal solutions:

K -equilibrium points and K -acceptable outcomes.

Let K be an arbitrary family of coalitions in game G .

Definition 1. A strategy $x_T^0 \in X_T$ is called a refutation of coalition T against outcome a if for any strategy $x_{N\setminus T} \in X_{N\setminus T}$ the condition $a \overset{\rho_T}{<} F(x_T^0, x_{N\setminus T})$ holds. An outcome a is called K *-acceptable* if any coalition $T \in K$ does not have a refutation against this outcome.

Definition 2. A strategy $x_T^0 \in X_T$ is called a refutation of coalition T against situation $x \in X$ if the condition $F(x) \overset{\rho_T}{<} F(x_T^0, x_{N\setminus T})$ holds. A situation $x^0 \in X$ is called K - *equilibrium* if any coalition $T \in K$ does not have a refutation against this situation.

3. In this section we find some connections between optimal cooperative solutions of games which are in homomorphic relations.

Let $\Gamma = \langle (U_i)_{i\in N}, B, (\sigma_i)_{i\in N}, \Phi \rangle$ be one more game of the same players and $f = (\phi_1, \ldots, \phi_n, \psi)$ be a homomorphism from game G into game Γ .

Definition 3. A homomorphism f is said to be:

• *a coalition homomorphism* if it preserves preference relations for coalitions, i.e. for any coalition $T \subseteq N$ the condition

$$a_1 \overset{\rho_T}{\lesssim} a_2 \Rightarrow \psi(a_1) \overset{\sigma_T}{\lesssim} \psi(a_2)$$

holds;

• *a strict coalition homomorphism* if for any coalition $T \subseteq N$ the system of the conditions

$$\begin{cases} a_1 \overset{\rho_T}{<} a_2 & \Rightarrow \quad \psi(a_1) \overset{\sigma_T}{<} \psi(a_2), \\ a_1 \overset{\rho_T}{\sim} a_2 & \Rightarrow \quad \psi(a_1) \overset{\sigma_T}{\sim} \psi(a_2) \end{cases}$$

is satisfied;

• *a regular coalition homomorphism* if for any coalition $T \subseteq N$ the system of the conditions

$$\begin{cases} \psi(a_1) \overset{\sigma_T}{<} \psi(a_2) & \Rightarrow \quad a_1 \overset{\rho_T}{<} a_2, \\ \psi(a_1) \overset{\sigma_T}{\sim} \psi(a_2) & \Rightarrow \quad \psi(a_1) = \psi(a_2) \end{cases}$$

is satisfied.

The main result of this paper is finding of a correspondence between sets of K-acceptable outcomes and K-equilibrium situations of games which are in homomorphic relations of indicated types.

References

[1] Rozen, V.V. (2009). Cooperative Games with Ordered Outcomes. Game Theory and Management. Collected abstracts of papers presented on the Third International Conference Game Theory and Management. SPb.: Graduate School of Management SPbU, pp. 221—222

Determining of Optimal Strategies via Coalitions using the Shapley Values

Sergei Schreider[1], **Panlop Zeephongsekul**[2] and **Babak Abbasi**[3]

[1,2,3]*School of Mathematical and Geospatial Sciences,*
RMIT University, Melbourne,
Australia
[1]*Sergei.Schreider@rmit.edu.au*
[2]*panlopz@rmit.edu.au*
[3]*b.abbasi@gmail.com*

Keywords: *Cooperative game, Fertilisation strategies,Utility function*

This paper continues the research described in the previous paper presented in GTM2009 [1] which describes the application of game theoretic approach in resource management with specific application to development of optimal strategies of phosphorus applications for soil fertilisation. In [1] we considered not only competitive strategies, which were treated as the Nash equilibrium game solutions but also strategies which imply cooperation between farmers. These strategies were modelled using the cooperative Pareto optimality concept. The paper presents algorithms for finding competitive and cooperative solutions of the game for the particular case when no time scheduling is included in the game parameterisation. The results obtained in [1] showed that the cooperative solutions lead to much lesser negative environmental impacts than in the case of non-cooperative strategies. This paper allowed us to improve these results by using coalitional strategies.

The major problem associated with this research was related to the fact that set of Pareto optimal solution for this game of 30 players (farmers) was too restrictive. In our model Pareto optima were obtained by maximising the weighted sum of player payoff functions where we have assumed that the weights are equal. Such an approach provided us with a better phosphorus management strategy compared to Nash equilibrium solution in most but not for all farmers. The major question remained how to find the 'better' cooperative optima, which will secure strict domination over Nash equilibrium solutions for all players? In the present paper, we employed the Shapley value calculation technique in order to address this problem.

Let u_i be the utility function of each player (refer to [1] for the definition of this and other related functions). In order to find Nash values, the following equations must be solved:

$$\frac{\partial u_i}{\partial \alpha_i^r} = 0\,,\; i = 1,...,N\;;\; r = 1,...,R$$

In order to find a Pareto optimum, the following equations must be solved:

$$\frac{\partial(\sum_{i=1}^{N} u_i)}{\partial \alpha_i^r} = \frac{\partial Z}{\partial \alpha_i^r} = 0\,,\; i = 1,...,N\;;\; r = 1,...,R$$

Z is the total weighted utility of all players and the Pareto optimum presents the maximum total utility of all player when they cooperate together.

The next stage in solving a cooperative game theory problem is in seeking a fair distribution of the gain to each player. It is clear that if individual utility values based on the cooperative strategy are assigned as gain to each player, some of the player will be worse off when compare with non-cooperative strategy. Therefore, in this case there is no rationale for them to cooperate. To calculate a fair allocation of total gain in a cooperative way, we use Shapley values, which are well known to have attractive property of a fair allocation. The Shapley value to Player i among the set of players

N ={1,2,…,n} calculated by using the following equation:

$$\Phi_i(\vartheta) = \sum_{S \subseteq N-\{i\}} \frac{\left|S\right|!(\left|N\right| - \left|S\right| - 1)!}{\left|N\right|!}[\vartheta(S \cup \left\{i\right\}) - \vartheta(S)]\,, i = 1,2,...,n \qquad (1)$$

where N is the set of players, S is a subset N and $\vartheta\cdot(\cdot)$ is the characteristic function of the cooperative game. Characteristic functions are often assumed to be super-additive, i.e. they satisfy

1. ϑ (∅) =0 and

2. ϑ (A∩B)≥ ϑ (A) + ϑ (B) for any mutually exclusive sets A and B, i.e. A∪B=∅.

To calculate the Shapely values for n players involves $2^n - 1$ pieces of computations. In our problem we have 30 players, so to calculate Shapely values for all players from the formula would involve 230-1 (10727*105) computations. We use MATLAB program in order to perform this huge number of computations and use them to obtain Shapley values.

A program in MATLAB 7 was written to calculate Shapely values considering above notes and it was run with (run time was 637005*108 sec. for the computer with the following characteristics: Intel 2 CPU, 1.83 GHz with 512 MB Memory RAM.

The Nash, Pareto and Shapley values were calculated for all 30 players of our model. The Pareto optima dominated over Nash equilibria for 27 out of 30 players, however three players earned less utility when the Pareto approach is applied. However, the total objective function, summarised for all players, of Pareto solution is greater than Nash one. The results for Shapely values calculated for each player indicated total domination of Shapley of Nash solutions: all players earned more utility compare to Nash values, due to the individual rationality as property of Shapely values. The major conclusion we made is that coalitional strategies provide more optimal managerial option than those associated to the Pareto cooperative solutions.

References

[1]. Schreider, S., Zeephongsekul, P. and Fernandes, M. 2009, A Game Theoretic Approach for Selecting of optimal Strategies of Fertiliser Application, In: Contribution to Game Theory and Management, Volume II, Petrosjan, L.A. and Zenkevich, N.A. (editors), Graduate School of Management, St. Petersburg University, pp. 415 – 436.

МАТЕМАТИЧЕСКАЯ ТЕОРИЯ ИГР И ЕЕ ПРИЛОЖЕНИЯ

Выходит ежеквартально с января 2009 года

Pure-Play or Multi-Channel Distribution: Which Market Structure Results in Equilibrium?

Marina T. Schroeder

University of Magdeburg,
Germany
marina.schroeder@ovgu.de

Keywords: *Channels of distribution, E-commerce, Game theory, Market entry*

The evolution of the internet brought about new distribution opportunities. Nowadays all companies face the decision whether to sell products or services online, offline or through both channels. This work studies the influencing factors for which multi-channel or pure-play distribution result as equilibrium market structures.

A game theoretic model is derived to predict the market equilibrium given the possibility of distributing products through the offline and the online channel. Under the assumptions that channels are imperfect substitutes and that distribution costs differ between channels, the distributors' market entry is modeled as a simultaneous decision problem. The model is applied to both a vertically integrated and a vertically separated distribution setting.

In the vertically integrated setting it can be shown, that equilibrium market structure is dependent on the cost difference and the level of channel competition. Either exclusively the more efficient channel is served, resulting in a pure-play equilibrium, or multiple distribution channels will coexist.

In the vertically separated setting a product can be sold through a monopolistic online retailer and a monopolistic offline retailer. Independent of differences in distribution costs, both retailers will decide to enter into sales as long as profits are non-negative. When deciding on market entry, the two retailers account for their own profits resulting from the corresponding post-entry competition. Retailers do not give consideration to external effects of their market entry. If substitutability of channels is high this may lead to an inefficient market entry of the retailer facing higher distribution costs. In such cases, coordination tools, enforcing exclusion of the disadvantaged retailer, yield overall efficiency gains. Thus the equilibrium market structure for vertically separated settings depends on whether such coordination tools are implemented or not.

Overall it is found that the equilibrium market structure depends on multiple factors. First, it depends on the number of intermediary levels included in the distribution process and on the level of coordination between these levels. Second, channel specific parameters such as distribution costs and channel substitutability have a major impact on the equilibrium market structure. It can be shown that due to consumers' channel preferences, the coexistence of multiple distribution channels is possible even if one channel faces higher distribution costs than the other.

МАТЕМАТИЧЕСКАЯ ТЕОРИЯ ИГР И ЕЕ ПРИЛОЖЕНИЯ

Ответственный редактор:
Л.А. Петросян
Зам. ответственного редактора:
В.В. Мазалов
Ответственный секретарь:
Н.А. Зенкевич

Выходит ежеквартально с января 2009 года

A Numerical Method to Compute Strategies for Some Differential Games

Sriram Shankaran[1], **Dusan Stipanovic**[2] and **Claire Tomlin**[3]

[1]*General Electric Research Center*
United States
sshankaran@gmail.com

[2]*University of illinois at Urbana-Champagne*
United States
dusan@uiuc.edu

[3]*EECS University of California at Berkeley*
United States
tomlin@eecs.berkeley.edu

Keywords: *Numerical Methods, Differential Games, Optimal Strategies*

Abstract: *In this paper, we use a numerical method to determine control laws for a few pursuit-evasion games. We consider two and three player games that have ''switched" control laws. We determine the optimal control inputs for the static HJI equation for the time-to-reach function is solved using the Gauss-Seidel (GS) Sweeping method proposed by Kao, Osher and Qian (Journal of Computational Physics, 2004). We provide theoretical results to qualify the computed value functions (and optimal inputs) and discuss higher-order approximations to reduce computational cost and multigrid algorithms to improve the rate of convergence. From the game- heoretic solution, trajectories for the various initial conditions are simulated to determine control laws for the players. For the two player game, the switched control laws result from the presence of universal surfaces. For the three player game, the switched behavior of the overall game is characterized by a switching surface composed of initial conditions of the evader, relative to the pursuers. Depending on the relative positions of the players, the game degenerates to either a game between one pursuer and the evader (a one-on-one game) or a game with both pursuers involved in the pursuit (a two-on-one game). Hence, the switched control laws for the pursuers can be represented through a combination of a logical input, that determines if the pursuer is involved in the pursuit and continuous control inputs that define the trajectory that the pursuers follow to capture the evader.*

The Continuously Stable Strategy (CSS) and Neighborhood Invader Strategy (NIS) concepts, originally developed as intuitive static conditions to predict the dynamic stability of a monomorphic population, are shown to be closely related to classical game-theoretic dominance criteria when applied to continuous strategy spaces. Specifically, for symmetric and non symmetric two-player games, a CSS in the interior of the continuous strategy space is equivalent to neighborhood half-superiority while an NIS is equivalent to full neighborhood superiority. These conditions generalize risk dominance and p^* -dominance concepts for two-strategy two-player games.

The CSS and NIS are also important for dynamic stability under the replicator and best response dynamics as well as for adaptive dynamics. In particular, these dominance criteria applied to two-species models give a game-theoretic method to predict the equilibrium behaviour of interacting populations.

In this paper we explore rent seeking contests (see Nitzan (1994) and Tollison (1997)), where the set of players is weak heterogeneous concerning their preferences. The players have negatively interdependent preferences in different parameter values. For a long time the assumption of preferences being independent from other players' payoff has been unquestioned in economic. In the more recent literature especially the literature about evolutionary game theory it became more common to relax this assumption of purely self-interested preferences. Hehenkamp et al. (2004) show that in rent-seeking contests, as analyzed here, it is evolutionary stable to behave like a player with interdependent preferences. Nevertheless the idea of interdependent preferences is rather old. Duesenberry (1949) laid a first empirical basis of interdependent preferences. A game-theoretic approach for the design of switched control laws was formulated in the context of hybrid systems by Tomlin, Lygeros and Sastry [1]. The discrete and continuous control inputs were derived using a two player zero-sum game theory formulation with the controller pitted against an unknown disturbance. The discrete inputs are determined from a discrete Hamilton-Jacobi (HJ) equation and the continuous inputs from the time-dependent Hamilton-Jacobi-Isaacs (HJI) equation. Applications to problems in aircraft conflict resolution, aerodynamic envelope protection and vehicle collision avoidance were presented along with computational tools based on numerical and other approximation methods. This game-theoretic formulation can also be used for differential games. One approach to this problem is to computationally solve the appropriate HJ equation and determine switched control laws (if they exist) from the solution. It is the use of this approach for differential game with two and three players that is the focus of this study. The static HJI equation for the time-to-reach problem is solved using a numerical method and control laws are extracted from the game-theoretic solution.

In this paper, we use a numerical method to determine control laws for a few pursuit-evasion games. We consider two and three player games that have "switched" control laws. We determine the optimal control inputs for the static HJI equation for the time-to-reach function is solved using the Gauss-Seidel (GS) Sweeping method proposed by Kao, Osher and Qian [2]. We provide theoretical results to qualify the computed value functions (and optimal inputs) and discuss higher-order approximations to reduce computational cost and multigrid algorithms to improve the rate of convergence. From the game-theoretic solution, trajectories for the various initial conditions are simulated to determine control laws for the players. For the two player game, the switched control laws result from the presence of universal surfaces. For the three player game, the switched behavior of the overall game is characterized by a switching surface composed of initial conditions of the evader, relative to the pursuers. Depending on the relative positions of the players, the game degenerates to either a game between one pursuer and the evader (a one-on-one game) or a game with both pursuers involved in the pursuit (a two-on-one game). Hence, the switched control laws for the pursuers can be represented through a combination of a logical input, that determines if the pursuer is involved in the pursuit and continuous control inputs that define the trajectory that the pursuers follow to capture the evader.

The algorithm to implement the GS Sweeping algorithm consists of three steps: 1) initialization of values within the computational domain, 2) GS Sweeping updates and 3) enforcing computational boundary conditions. For our differential game, there are either three or four continuous states and the computational grid is built from uniform discretizations of size Δx in each of the continuous states.

1. Assign a value to all the interior points in the computational grid which is larger than the true solution. These values will be updated by the sweeping algorithm.

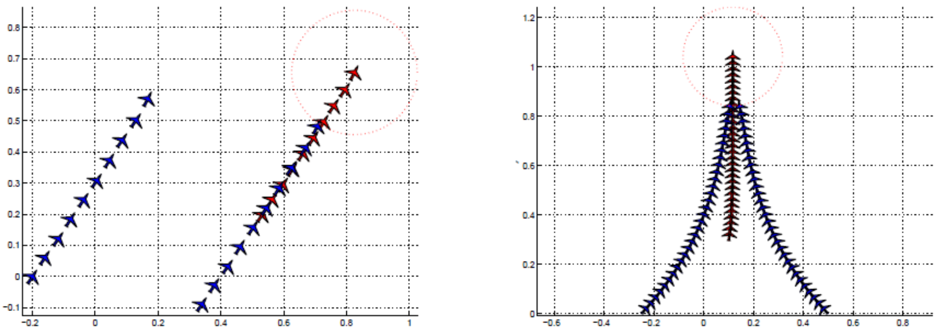
2. During each iteration, use the GS Sweeping formula, to update the values in the computational grid. We use 8 or 16 sweeps for the three or four dimensional computational grid in our problems. Each sweep consists of a forward or backward sweep of a particular coordinate direction while keeping the direction of the sweep in the other directions fixed.

3. Enforce boundary conditions to update values on the edges of the computational grid.

4. Repeat steps 2 and 3 until the maximum change in the value at any point within the computational grid is less than a predefined threshold, e .

In the final version of the paper, we will show that the value functions are viscosity solutions. In addition, we will discuss the use of higher-order approximations to reduce computational costs and multigrid algorithms to improve convergence of the GS Sweeping algorithm.

Sample optimal trajectories for a three player game are shown in Fig. 1(a) and 1(b).



(a) One-on-One Game (b) Two-on-One Game

Figure 1: Bifurcation of the game space leading to one-on-one and two-on-one games

References

[1] Tomlin C., Lygeros J., and Sastry S.. A Game-Theoretic Approach to Controller Design for Hybrid Systems Proceedings of the IEEE, Volume 88, Number 7, July 2000.

[2] Kao. C.Y., Osher S. and Qian J., Lax-Friedrichs sweeping scheme for static Hamilton-Jacobi equations, Journal of Computational Physics, Vol. 196, Issue 1, pp. 367-391, 2004.

A Fuzzy Cooperative Game Model for Open Supply Network Configuration Management

Leonid Sheremetov[1] and **Alexander Smirnov**[2]

SPIIRAS,
Russian Federation
[1]*lsheremetov@mail.ru*
[2]*smir@iias.spb.su*

Keywords: *Fuzzy cooperative game, Coalition formation, Configuration management*

Abstract: *A fuzzy cooperative game model for coalition formation is developed and applied for configuring of supply networks. See the file attached.*

In a today's highly competitive market manufacturers face the challenge of reducing manufacturing cycle time, delivery lead-time and inventory. As a consequence, new organizational forms of enterprise integration emerge to address these challenges resulting in more agile structures known as adaptive, agile and open supply chains and networks. In contrast to conventional chains and supply networks, open supply networks (OSNs) are characterized by the availability of alternative providers and the expediency of dynamic configuration and reconfiguration of the network depending on the order stream and economic benefit of every enterprise. A possible form of this flexible configuration consists in generation of enterprise coalitions.

Unfortunately, adoption of more flexible and dynamic practices, which offer the prospect of better matches between suppliers and customers as market conditions change, has proven elusive, due to the complexity of many supply chain relationships and the difficulty of effectively supporting dynamic trading practices. Due to the conflictions among the objectives of each organization and non-integrated decision making processes, there has been a need for new coalition formation mechanisms, which help to resolve those conflictions and to agree upon effective solutions.

Coalition formation represents a rapidly developing field of research. One of the possible approaches to coalition formation is represented by the theory of games [1]. Till now, in many research efforts non-cooperative games have been applied to model supply chains and the interaction of the enterprises as a zero-sum game. In that context, all the players are considered being self-interested trying to optimize their own profits without taking into account their effects on the other players. The main problem is to find the optimal strategy for each player. On the contrary, the theory of cooperative games offers results that show the structure of possible interaction and the conditions required for it. In many cases, negotiations and bargaining take place before any concerted strategy is used, when the players have a vague idea of what the expected coalition benefit will be. Thus, profit distribution can prove to be fuzzy, uncertain, and ambiguous [2]. Using the theory of fuzzy cooperative games (FCGs), we can process the uncertainty and pass from the introduction of a fuzzy profit concept through the bargaining process to the conclusion about the corresponding fuzzy distribution of individual payoffs.

To model the OSN configuration process, in this paper fuzzy coalition games with a solution set are considered, more particularly games with associated core [3]. The core of a game with respect to a given coalition structure is defined as a set of coalition configurations that don't necessarily have unique payoff distributions. Those coalition structures that maximize the total profit (i.e. the sum of all coalition values in the considered structure) are called coalitions with a stable core (C-stable), or effective coalitions. Unfortunately, games with the core have well-known drawbacks: the core might be empty for certain cooperative games and is exponentially difficult to compute. Because of these problems, using the C-stable coalition has been quite unpopular so far. In this paper we show that these problems can be solved in a proposed generalized model of a FCG.

A FCG is defined as a pair (I,w), where I is nonempty and finite set of players, subsets of I joining together to fulfil some task t_j are called coalitions, and w is called a characteristic function of the game, being $w{:}2^I \to R^+$ a mapping connecting every coalition $K \subset I$ with a fuzzy quantity $w(K) \in R^+$, called characteristic function of the game, with a membership function $\mu_K : R \to [0,1]$, where a characteristic function of the crisp game $v(K)$ is the modal value of $w(K)$ and for the empty coalition $w(\varnothing) = 0$. The solution of a cooperative game is a coalition configuration (S, x) which consists of (i) a partition S of I, the so-called coalition structure, and (ii) an efficient payoff distribution x which assigns each agent in I its utility out of the value of the coalition it is member of in a given coalition structure S. A coalition configuration (S, x) is called stable if no agent has an incentive to leave its coalition in S due to its assigned payoff xi. Let us define a fuzzy core for the game (I, w) with the imputation $X = (x_{ij})_{i\in I, j\in J} \in R^{+}$ as a fuzzy subset CF of R^{+}:

$$C_F = \left\{ x_{ij} \in R^{+} : \nu \succ= (w(I), \sum_{\substack{i\in I,\\ j\in J}} x_{ij} y_{ij}), \min_{\substack{K_j \in \bar{k}\\ j\in J}} (\nu \succ= (\sum_{i\in K_j} x_{ij} y_{ij}, w(K_j)) \right\} \tag{1}$$

where x_{ij} is the fuzzy payment of an agent i participating in a coalition, $j, i = 1,2,...,n$, $j = 1,2,...,l$, $\bar{k}$= [K1, K2, ..., Kl] is the ordered structure of effective coalitions; y_{ij} - is a *fuzzy partial order relation* with a membership function $\nu \succ= : R \times R \to [0,1]$, and

$$y_{ij} = \begin{cases} 1, & \text{if an agent } i \text{ participates in a coalition } j; \\ 0 & \text{otherwise.} \end{cases}$$

The core C_F is the set of possible distributions of the total payment achievable by the coalitions, and none of coalitions can offer to its members more than they can obtain accepting some imputation from the core. The first argument of the core C_F indicates that the payments for the grand coalition are less than the characteristic function of the game. The second argument reflects the property of group rationality of the players, that there is no other payoff vector, which yields more to each player. The membership function $\mu_{C_F} : R \to [0,1],$ is defined as:

$$\mu_{C_F}(X) = \min \left\{ x_{ij} \in R^{+} : \nu \succ= (w(I), \left\langle \sum_{\substack{i\in I,\\ j\in J}} x_{ij} y_{ij} \right\rangle), \min_{\substack{K_j \in \bar{k}\\ j\in J}} (\nu \succ= (\left\langle \sum_{i\in K_j} x_{ij} y_{ij} \right\rangle, w(K_j)) \right\} \tag{2}$$

A game (I, w) may be superadditive, subadditive, or simply additive. The properties of the game are defined in three lemmas and two theorems. One of them proves that the fuzzy set of coalition structures forming the game core represents a subset of the fuzzy set formed by the structure of effective coalitions. In turn, this inference allows us to specify the upper possibility bound for the core, which is a very important condition for the process of solution searching, because in this case, the presence of a solution that meets the efficiency condition may serve as the signal to terminate the search algorithm.

Theorem. Let (I, w) be a fuzzy coalition game. Then for some structure of effective coalitions $\bar{k}$, its possibility is at least equal to the possibility of forming the core.

It is necessary to note that the statements take into account only the characteristics of the game (I, w); therefore, any real argument can be introduced into the fuzzy core. For example, such restrictions as a number of agents in each coalition and those defining coalitions to be overlapping or not or regulating the tasks order are admissible. This feature is very important for the application of the model for OSN configuration management.

To find the analytical (exact) solution of the FCG, it is necessary to determine the fuzzy super-optimum and the fuzzy relation of domination [2], which is extremely difficult in real applications. Therefore, it is proposed to use a heuristic technique of finding solutions that are close to the optimal one. In the considered case, the techniques of soft computing using genetic algorithms (GA) in the context of fuzzy logic are applied. It is equivalent to binary encoding of the fuzzy core with the fitness function equal to the supremum of all minimums of the membership function. Application of GA allows one to obtain an approximate solution for the games with a large number of players and a membership function of any type. Being an anytime algorithm that steadily improves the solution, the GA can find the best solution under the time constraints.

In the conducted experiments on model complexity the number of iterations needed to approach the optimal solution served as the investigated variable with the following factors: the number of agents and coalitions, the accuracy, and the order of fuzzy payments. Results show that the number of iterations (computation time) decreases or remains constant when the number of agents increases. In other words, it takes less time to form coalitions. On the other hand, the results demonstrate almost linear relation between the numbers of coalitions and agents. On the whole, the experiments justified that all factors are highly significant; the only surprise was that the order of fuzzy payments substantially influence the number of iterations (the convergence time).

To apply the proposed model in real life scenarios, a multiagent software environment was developed consisting of three types of agents: individual, coalition and integrator. Agents are associated with the OSN as follows: the integrator agent simulates original equipment manufacturer (OEM); the coalition agents are responsible for component production and assembling; and the individual agents represent row material suppliers and component producers. The model was applied for configuring of a 3-echelons automotive OSN taking into account that an enterprise may belong to different echelons (the case of overlapping coalitions). The simulation results are considered.

References

[1]. J. P. Aubin, Cooperative Fuzzy Games, Math. Oper. Res. 6 (1), 1-13, 1981.
[2]. M. Mares, Fuzzy Cooperative Games - Cooperation with Vague Expectations, Physica Verlag, 2001.
[3]. Gillies, D.B. Some theorems on n−person games. Ph. D. thesis, Princeton University Press. Princeton, New Jersey, 1953.

Guaranteed Pursuit Strategies for the Games with Symmetric Terminal Alternatives

Igor Shevchenko

Head of IT
Russian Federation
igor@tinro.ru

Keywords: *Alternative pursuit games, Feedback approximation, Chatter*

Abstract: *In the paper we will discuss how to modify the guaranteed pursuit strategies in order to avoid a chatter.*

Let the state $\mathbf{z}(t)$ in the playing space $\mathbf{Z} \subseteq \mathbb{R}^n$ obey the equation $\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}, \mathbf{u}_p, \mathbf{u}_e)$, $\mathbf{z}(0) = \mathbf{z}^i$. The pursuer P and the evader E choose their controls $\mathbf{u}_p(t)$ and $\mathbf{u}_e(t)$ from U_p and U_e, which are compact convex sets in $\mathbb{R}^{n_p}$ and $\mathbb{R}^{n_e}$, $t \geq 0$. A pursuit game B is called *alternative* if it may be terminated by P on any of two smooth terminal hypersurfaces M_a and M_b, and corresponding payoffs M_a and M_b of Boltza type have the same integrand L_{ab} and differ only in their terminal parts K_a and K_b [1]. Assumed that $\mathbf{f}, L_{ab}$, K_a and K_b are at least twice continuously differentiable functions in respective domains. We associate with B two games B_a and B_b that correspond to the fixed alternatives, a and b, and have M_a and M_b as the terminal surfaces, T_a and T_b as the terminal instants, and P_a and P_b as the payoffs. For a given initial state $\mathbf{z}^i \in \mathrm{Z}$ and a pair of admissible strategies (S_p, S_e), the payoff in B equals $P_{k(\mathbf{z}^i, S_p, S_e)}$ where $T_{k(\mathbf{z}^i, S_p, S_e)} = \min\left(T_a(\mathbf{z}^i, S_p, S_e), T_b(\mathbf{z}^i, S_p, S_e)\right)$, $k(\mathbf{z}^i, S_p, S_e) \in L = \{a, b\}$. Also, let $B_{a|b}$ be the game with the terminal manifold $\mathrm{M}_{a|b} = \mathrm{M}_a \cup \mathrm{M}_b$ and the payoff $P_{a|b}$ equal to $P_{m(\mathbf{z}^i)}$ where $m(\mathbf{z}^i) \in L = \{a, b\}$ is assigned by P at the initial instant.

Suppose that solutions for B_a and B_b are known. Let S_p^l and S_e^l be the optimal strategies of the players, $V^l(\mathbf{z})$ be the value function, $\mathbf{z}_l(t, \mathbf{z}^0)$ be the optimal trajectory and $\tau_l(\mathbf{z})$ be the optimal duration in B_l for $\mathbf{z} \in \mathrm{Z}$, $l \in L$. The value function $V^{a|b}$ of $B_{a|b}$ depends on the current $\mathbf{z}$ and initial $\mathbf{z}^i$ states and is described as follows $V^{a|b}(\mathbf{z},\mathbf{z}^i) = V^{m(\mathbf{z}^i)}(\mathbf{z})$, where $m(\mathbf{z}^i) \in L$ is chosen by P for the whole duration of the game such that $V^{m(\mathbf{z}^i)}(\mathbf{z}^i) = \min\left(V^a(\mathbf{z}^i), V^b(\mathbf{z}^i)\right)$.

Let D^0 be the hypersurface of codimension $n-1$ where both alternatives are of the equal value, $\mathrm{D}^0 = \{\mathbf{z}^i : V^a(\mathbf{z}^i) = V^b(\mathbf{z}^i)\}$, and Z_a^i be the subset where a dominates b, $\mathrm{Z}_a^0 = \{\mathbf{z}^i : V^a(\mathbf{z}^i) < V^b(\mathbf{z}^i)\}$. We distinguish a *negligible violation* of the initial dominating condition for a along the optimal trajectory of B_a when $V^a(\mathbf{z}^i) \le V^b(\mathbf{z}^i)$ and $\exists t > 0 : V^a\left(\mathbf{z}_a(t,\mathbf{z}^i)\right) = V^b\left(\mathbf{z}_a(t,\mathbf{z}^i)\right)$, and a *substantial violation* when

$$\exists \varepsilon > 0\ \exists t' \in [0, \tau_a(\mathbf{z}^i) - \varepsilon]\ \forall t \in [t', t' + \varepsilon] : V^a\left(\mathbf{z}_a(t,\mathbf{z}^i)\right) > V^b\left(\mathbf{z}_a(t,\mathbf{z}^i)\right).$$

Let D_a be the subset of D^0 where the initial dominating condition for a is stable, and $\mathrm{D}_{\bar{a}}$ be the subsets of D_a^0 where it is violated substantially. Let Z_a be the subset of Z_a^0 where the initial dominating condition for a is stable, and B_a and $\mathrm{Z}_{\bar{a}}$ be the subsets of Z_a^0 where it is violated negligibly and substantially. Define D_b, $\mathrm{D}_{\bar{b}}$, B_b, $\mathrm{Z}_{\bar{b}}$ similarly.

In a game B with *symmetric terminal alternatives*, D_a and D_b, and $\mathrm{D}_{\bar{a}}$ and $\mathrm{D}_{\bar{b}}$ coincide. Denote them hereafter as $\mathrm{D}_{a|b}$ and $\mathrm{D}_{\overline{a|b}}$. Also, there exist the set $\mathrm{A} \subset \mathrm{D}_{a|b}$ of common limiting states of B_a and B_b where the time derivative of $V^a(\mathbf{z}) - V^b(\mathbf{z})$ equals zero along the both optimal trajectories with fixed alternatives. Assume that P plays with S_p^a and S_p^b in Z_a and Z_b and, therefore, gains at least $V^{a|b}(\mathbf{z}) = \min\left(V^a(\mathbf{z}), V^b(\mathbf{z})\right)$ there. We call a pursuit strategies *guaranteed* if it provides a payoff less than $V^{a|b}(\mathbf{z})$ for all $\mathbf{z} \in \mathrm{Z}_{\overline{a|b}} = \mathrm{Z}_{\bar{a}} \cup \mathrm{Z}_{\bar{b}} \cup \mathrm{D}_{\overline{a|b}}$ where P doesn't fix a particular terminal alternative. It turns out that P must target A as a transitional hypersurface there, and along a potentially optimal trajectory the state cannot arrive at A bypassing $\mathrm{D}_{\overline{a|b}}$ [1, 2].

A solution on $\mathrm{D}_{\overline{a|b}}$, which is a hypersurface of codimension $n-1$, can be obtained only when E keeps the states there using a discriminating strategy, which involves instantaneous knowledge of P's control. Feedback approximations of a constructed evasion strategy in $\mathrm{Z}_{\overline{a|b}}$ may lead to a chatter [3, 4]. In the paper we will discuss how to modify the guaranteed pursuit strategies in order to avoid a chatter along $D_{\overline{a|b}}$.

References

[1] Shevchenko, I., On Reduction of Alternative Pursuit Games, Game Theory and Applications, 11, 125 - 137 (2004).
[2] Shevchenko, I., Strategies for Alternative Pursuit Games. In P. Bernhard, V. Gaitsgory, O. Pourtallier (Eds.) Advances in Dynamic Games Theory and Their Applications: Analytical and Numerical Developments, Annals of the International Society of Dynamic Games, 10, Boston, Basel, Berlin, Birkhauser, 121 - 131 (2009).
[3] Filippov, A. F., Differential Equations with Discontinuous Right-hand Sides, Kluwer, Dordrecht (1988).
[4] Breakwell, J.V., Bernhard, P., A Simple Game with a Singular Focal Line, Journal of Optimization Theory and Applications, 64, 2, 419 - 428 (1990).

Modeling of Environmental Projects under Condition of a Random Game Duration

Ekaterina Shevkoplyas[1] and **Sergey Kostyunin**[2]

[1,2]*St.Petersburg State University*
Russian Federation
[1]*katya_shev@mail.ru*
[2]*isg23@yandex.ru*

Keywords: *Differential games, Environment, Random duration.*

Abstract: *One game-theoretical model of environmental project is considered under condition of a random game duration. The game ends at the random moment in time with Weibull distribution. According to Weibull distribution form parameter, the game can be in one of 3 stages such as "an infant" stage, "an adult" stage and "an aged" stage. The solutions obtained with a help of Pontryagin maximum principle both for non-cooperative and cooperative forms of the game are analyzed for each stage.*

The game-theoretical model of environmental project [1] with 2 non-identical players had been considered under condition that the game has a fixed terminal time T. We consider the modification of this model with elements of stochastic framework, in the sense that the terminal time T is a random value [2-4] in order to increase the realness of the modeling. It turns out that this formulation of the game can be simplified to standard formulation for dynamic programming. With the help of Pontryagin maximum principle we find the solutions both for non-cooperative and cooperative forms of the game under condition of Weibull distribution for the random value T [4]. According to Weibull distribution form parameter, the game can be in one of 3 stages such as "an infant" stage, "an adult" stage and "an aged" stage. The solutions are analyzed for each stage of the game and the interpretation in the context of the environmental economics is given.

References

[1] Breton M., Zaccour G., Zahaf M. A Differential Game of Joint Implementation of Environmental Projects. Automatica, Volume 41, Issue 10, 2005, pp.37-1749
[2] Petrosjan L.A., Murzov N.V. Game-theoretical Problems in Mechanics. Lithuanian Mathematical Collection. VI-3, 1966. pp.423-433 (in Russian).

[3] Petrosjan L.A., Shevkoplyas E.V. Cooperative Solutions for Games with Random Duration. Game Theory and Applications, Volume IX. Nova Science Publishers, 2003, pp.125-139.
[4] Shevkoplyas E.V. The Hamilton-Jacobi-Bellman Equation in Differential Games with Random Duration. Upravlenie bolshimi systemami, 26.1, M.: IPU RAN, 2009, pp.385-408 (in Russian).

Endogenous and Exogenous Switching Costs: Complements or Substitutes?

Mengze Shi

University of Toronto
Canada
mshi@rotman.utoronto.ca

Keywords: *Switching cost, Game theory, Customer loyalty, Marketing*

Switching costs refer to the costs that buyers have to incur when switching suppliers. There are two types of switching costs: exogenous and endogenous switching costs. The endogenous switching costs arise from firms' marketing decisions and hence the size of endogenous switching costs is determined by the firms. In contrast, the exogenous switching costs are not under firms' control. (An extensive review of both types of switching costs is provided by Farrell and Klemperer (2007).) The objective of this paper is to study the relation between exogenous and endogenous switching costs. We develop a two-period symmetric duopoly model where both competitors decide the size of endogenous switching costs in the form of future rewards according to the exogenous switching costs and the extent of horizontal differentiation (measured by the unit transportation cost in a Hotelling model). We find that the equilibrium endogenous switching costs decreases with the size of exogenous switching costs but increases with the extent of differentiation. While both types of switching costs help retaining customers, the endogenous switching costs also help acquire customers. Interestingly the efficiency of using endogenous switching costs as a customer-acquisition tool decreases with the exogenous switching costs because a higher retention rate leads to more customers redeeming the rewards in the second period. Thus, as the exogenous switching costs decreases, the endogenous switching costs as a customer-acquisition mechanism becomes more efficient as compared to other acquisition tools such as first-period price. Consequently in the equilibrium we observe greater endogenous switching costs and higher first-period prices.

References

[1] Farrell, Joseph and Paul Klemperer (2007), "Coordination and Lock-in: Competition with Switching costs and network Effects" in Handbook of Industrial Organization, Vol. 3, edited by Mark Armstrong and Robert H. Porter, Elsevier, North Holland, 1967-2072.

The Generalized Nucleolus as a New Solution Concept of Cooperative TU-games

Nadezhda V. Smirnova[1] and **Svetlana I. Tarashnina**[2]

[1]*St.Petersburg State University,*
International Banking Institute,
Russia
nadezhda.v.smirnova@gmail.com

[2]*St.Petersburg State University,*
Russia
tarashnina@gmail.com

Keywords: *Cooperative games, The generalized nucleolus, The nucleolus, The SM-nucleolus*

Abstract: *In this paper we consider a new solution concept of cooperative TU-games, called as the generalized nucleolus. It is based on the ideas of the prenucleolus and the SM-nucleolus. We show that the generalized nucleolus of an arbitrary n–person TU-game coincides with the prenucleolus of a certain n–person constant-sum game, which is constructed as a weighted sum of a game and its dual. Also we discuss other properties of this solution.*

In the paper we describe a new solution concept of a cooperative TU-game (N, v), called the generalized nucleolus. It is based on the idea of the SM-nucleolus [5], some properties of which for different classes of cooperative game were studied in [6], [7]. The generalized nucleolus takes into account both the constructive power $v(S)$ and the blocking power $v^*(S)$ of coalition S with coefficients α and $1-\alpha$ accordingly with $\alpha \in [0, 1]$. A meaning of the constructive power $v(S)$ is clear. That is the worth of the coalition, or exactly what S can reach by cooperation. By the blocking power[1] of coalition S we understand the amount $v^*(S)$ that the coalition brings to N if the last will be formed – its contribution to the grand coalition. The value $v^*(S)$ is the difference between the amounts which can be received by the compliment $N \setminus S$ when it cooperates with S (in this case the grand coalition forms) and does not cooperate with S. In fact, if S leaves the grand coalition, then the complement coalition gets $v(N \setminus S)$.

[1] For the first time the blocking power of a coalition was taken into account in the Modiclus [4].

However, acting together they can receive $v(N)$. Thus, the difference between $v(N)$ and $v(N \setminus S)$ is a subject which should be taken into account in a solution of a game. In our opinion, the blocking power can be judged as a measure of a necessity of S for N – how much S contributes to N. So, each coalition S is estimated by N in this spirit. According to this motivation we suppose that a solution of a cooperative game should take into account the constructive power $v(S)$ and the blocking power $v^*(S)$. Then the following question occurs: in what ratio the constructive power and the blocking powers should be considered in a solution of a TU-game. In this paper, introducing the generalized nucleolus, we consider all possible ratios between the constructive and blocking powers.

In our work we show that for every fixed $\alpha \in \left[0, 1\right]$ the generalized nucleolus of an arbitrary n–person TU-game coincides with the prenucleolus of a certain n–person constant-sum game, which is constructed as the weighted sum of the game and its dual. Geometrically, the generalized nucleolus is a connected set of preimputations representing a sum of segments in R^n. This set has two extreme points: one of which corresponds to $\alpha = 1$, the other one corresponds to $\alpha = 0$.

Also in the paper we consider several well-known solutions of cooperative TU-games such as the Shapley value [3], the prenucleolus [2], the SM-nucleolus [5], the core [1]. It is shown that the generalized nucleolus and the prenucleolus coincide for arbitrary game (N, v) for $\alpha = 1$ (particularly, this indicates that for balanced games the intersection of the generalized nucleolus and the core is a nonempty set containing at least the prenucleolus). For $\alpha = \frac{1}{2}$ the generalized nucleolus coincides with the SM-nucleolus of a game. For three person cooperative TU-games we have the following curious result: the generalized nucleolus coincides with the Shapley value in case $\alpha = \frac{1}{2}$.

Finally, in order to illustrate some properties of the new solution we give several examples. In Example 1 we consider a five person cooperative game and construct the generalized nucleolus. It is easy to see that the generalized nucleolus coincides with the Shapley value for $\alpha = \frac{67}{80}$. Obviously, the generalized nucleolus coincides with the SM-nucleolus for $\alpha = \frac{1}{2}$ and the prenucleolus for $\alpha = 1$. Example 2 helps us to understand how the generalized nucleolus changes if we increase the constructive power of coalition $S=\{1,2,3\}$ as compared with the one in Example 1. On the top of this we apply the generalized nucleolus to the weighted majority games. In Examples 3, 4 we construct the generalized nucleolus and find the conditions under which the generalized nucleolus coincides with the Shapley-Shubik, Banzhaf and Johnston indices.

References

[1] Scarf, H. 1967. The core of an n-person game. Econometrica, (35): 56–69.

[2] Schmeidler, D. 1969. The nucleolus of a characteristic function game. SIAM Journal on Applied Mathematics, (17): 1163–1170.

[3] Shapley, L. S. 1953. A value for n-person games. In: Kunn and Tucker (eds.) Contributions to the Theory of Games, II. Princeton University Press, 307–317.

[4] Sudhölter, P. 1997. The Modified nucleolus: properties and axiomatizations. International Journal of Game Theory, (26): 147–182.

[5] Tarashnina, S., 2010. The simplified modified nucleolus of a cooperative TU-game. TOP. Special issue (accepted for publication).

[6] Tarashnina, S. 2007. The simplified modified nucleolus and the Shapley value. Materials of the III Spain Italy Netherlands Meeting on Game Theory (SING3).

[7] Tarashnina, S., Smirnova, N. 2008. The simplified modified nucleolus and its interconnection with other solutions of cooperative TU-games. The collection of the materials of the XXIX Conference "The Controlled Processes and Stability": 498-502.

Designing Strategies for Nonzero-Sum Differential Games using Differential Inequalities

Dušan M. Stipanović[1], **Emiliano Cristiani**[2] and **Maurizio Falcone**[2]

[1]*Department of Industrial and Enterprise Systems Engineering and Coordinated Science Laboratory, University of Illinois, Urbana, USA; dusan@illinois.edu*

[2]*Dipartimento di Matematica, SAPIENZA - Universita di Roma, Roma, Italy; cristian,falcone@mat.uniroma1.it*

Keywords: *Nonzero-sum differential games, Differential inequalities*

Abstract: *In this paper, we present an approach to design control strategies for multi-player nonzero sum differential games. In particular, we show how closed-form strategies for players in nonzero sum games can be formulated and designed using differential inequalities and the comparison principle. We will also show how the collaboration and the competition between the players can be incorporated into the players' goal functions and to illustrate the approach we consider problems related to multi-agent systems with multiple objectives such as trajectory tracking, coverage control, persistent surveillance, and collision avoidance.*

In this paper, we present an approach to design control strategies for multi-player nonzero sum differential games. Following ideas presented in [5] and [6] we show how closed-form strategies for players in nonzero sum games can be formulated and designed using differential inequalities [2, 3] and the comparison principle.

To approximate the minimum $a_{\min}$ of a set of nonnegative numbers $\{a_1, a_2, ..., a_n\}$ we recall the following functions:

$$\underline{\sigma}(\delta, a_1, \ldots, a_n) = \left(\sum_{i=1}^{n} a_i^{-\delta} \right)^{-1/\delta}, \ \overline{\sigma}(\delta, a_1, \ldots, a_n) = \left(n^{-1} \sum_{i=1}^{n} a_i^{-\delta} \right)^{-1/\delta} \tag{1}$$

which converge to $a_{\min}$ as δ increases. These functions satisfy that $\underline{\sigma}(\cdot)$ is a lower approximation and $\overline{\sigma}(\cdot)$ is an upper approximation of $a_{\min}$. Similarly, to approximate the maximum $a_{\max}$ of a set of nonnegative numbers $\{a_1, a_2, ..., a_n\}$ we recall the following functions:

$$\underline{\rho}(\delta, a_1, \ldots, a_n) = \left(n^{-1} \sum_{i=1}^{n} a_i^{\delta} \right)^{1/\delta}, \ \overline{\rho}(\delta, a_1, \ldots, a_n) = \left(\sum_{i=1}^{n} a_i^{\delta} \right)^{1/\delta}, \qquad (2)$$

which converge from below and above to the exact maximum as δ goes to infinity, respectively. In addition, using some known results in inequalities related to means [1, 4], we can show that the lower approximations $\underline{\sigma}(\delta, a_1, \ldots, a_n)$ and $\underline{\rho}(\delta, a_1, \ldots, a_n)$ are monotonically increasing functions of δ, while $\overline{\sigma}(\delta, a_1, \ldots, a_n)$ and $\overline{\rho}(\delta, a_1, \ldots, a_n)$ are monotonically decreasing functions of δ.

Let us first assume that the multiple objective functions in a nonzero-sum multi-player differential game are denoted by v_i, $i \in \{1, 2, ..., M\}$ where M denotes the number of objectives not necessarily equal to the number of players in the game denoted by N. In general the number of objectives is assumed to be larger than the number of players, that is, $M > N$. The objectives are satisfied if the trajectories of the differential equations modeling the behavior of the players either enter or stay away from the sets defined by objective functions as $\{v_i(x_i) \le q_i\}$ where x_i represents the state variables related to objective i and q_i's are appropriately chosen constants. The goal functions for each player will be denoted by w_p (the index p corresponds to player p) and their construction will use the approximations of the minimum and maximum given by (1) and (2). Note that, for each player, the goal functions will depend on the corresponding objective functions, that is, we have $w_p = w_p(v_1, v_2)$ if player p's objective functions are v_1 and v_2. This construction is formalized by establishing a link between the minimum and the maximum approximation functions with logical "and" and "or" functions according to the particular goals of the players. Once the goal functions are determined, the design of the players' strategies is based on a Liapunov-like approach involving differential inequalities and the comparison principle [5, 6]. For example, in the nonautonomous case where the dynamics of the players are given by

$$\dot{x}_p = f_p(t, x_p, u_p), \quad x_p(t_0) = x_{p0}, \ t_0 \in \mathbb{R}_+ = [0, +\infty), \quad p = 1, \ldots, N \qquad (3)$$

we will have a set of inequalities like

$$dw_p / dt \le G_p(t, x, w_p), \qquad (4)$$

for suitable functions $G_p(\cdot)$ that provide closed-form strategies $u_p(\cdot)$ and guarantee desired results based on the properties of the maximal solution of the differential equation $\dot{z}(t) = G_p(t, x(t), z)$, $z(t_0) = z_0$ [2]. The state variables associated with player p are gathered in vector x_p. We also use differential inequalities bounding time derivatives of goal functions from below and the corresponding minimal solutions to establish the accomplishment of objectives (see [6] for more details).

We will also show how the collaboration and the competition between the players can be incorporated into the goal functions. In this work, we consider problems related to multi-agent systems with multiple objectives such as trajectory tracking, coverage control, persistent surveillance, and collision avoidance.

Acknowledgement: This work has been supported by the Boeing Company via the Information Trust Institute, University of Illinois at Urbana-Champaign.

References

[1] P. S. Bullen, D. S. Mitrinović, and P. M. Vasić. Means and Their Inequalities, D. Reidel Publishing Company, Dordrecht, Holland, 1988.
[2] V. Lakshmikantham and S. Leela. Differential and integral inequalities: Theory and applications, Mathematics in Science and Engineering, vol. 55, Academic Press, New York, NY, 1969.
[3] V. Lakshmikantham, S. Leela, and A. A. Martinyuk. Stability analysis of nonlinear systems, Marcel Dekker, New York, NY, 1989.
[4] D. S. Mitrinović (with P. M. Vasić). Analytic Inequalities, Springer-Verlag, Berlin-Heidelberg, Germany, 1970.
[5] D. M. Stipanović, A. Melikyan, and N. Hovakimyan, Some Sufficient Conditions for Multi-Player Pursuit-Evasion Games with Continuous and Discrete Observations, Annals of the International Society of Dynamic Games, vol. 10, pp. 133-145, 2009.
[6] D. M. Stipanović, A. Melikyan, and N. Hovakimyan. Guaranteed Strategies for Nonlinear Multi-Player Pursuit-Evasion Games, to appear in the International Game Theory Review.

On Problems of Prediction and Optimal Desision Making for a Macroeconomic Model

Nina N. Subbotina[1] and **Timofey B. Tokmantsev**[2]

Institute of Mathematics and Mechanics,
UrB of RAS, Ekaterinburg,
Russia
[1]*subb@uran.ru*
[2]*tokmantsev@imm.uran.ru*

Keywords: *Optimal feedback, Non-antagonistic game, Nash equilibrium*

Abstract: *A macroeconomic model due to E.G. Al'brekht is under consideration. The model is described by two ordinary differential equations. The problem of identification for parameters of the model is solved as statistic data are given. The model is studied in order to get scenarios of a short-term prediction and to achieve optimal decision making for two participants of the market in an non-antagonistic game.*

A macroeconomic model due to E.G. Al'brekht is under consideration. The model is described by two ordinary differential equations. The problem of identification for parameters of the model is solved as statistic data are given. The model is studied in order to get scenarios of a short-term prediction and to achieve optimal decision making for two participants of the market in an non-antagonistic game.

A macroeconomic model due to E.G. Al'brekht [4] is under consideration. Symbol p denotes the gross product, q denotes production costs, h denotes profit. The function $G(p,q) = h$ is called the macroeconomic potential of the system. We assume that the function $G(p,q)$ has the polynomial form

$$G(p,q) = pq[a + f^{(1)}(p,q) + \ldots + f^{(m)}(p,q)],$$

where coefficients of the polynomials $f^{(0)} = a$, $f^{(1)}$, $\ldots$, $f^{(m)}$ have to be defined.

The statistic data of the form

$$(p^*(t_0), q^*(t_0), h^*(t_0)), (p^*(t_1), q^*(t_1), h^*(t_1)), \ldots, (p^*(t_N), q^*(t_N), h^*(t_N)).$$

are given.

The dynamics of the macroeconomic system are described by the following differential equations

$$\frac{dp}{dt} = u_1(t)\frac{\partial G(p,q)}{\partial p}, \ \frac{dq}{dt} = -u_2(t)\frac{\partial G(p,q)}{\partial q},$$

where $u_1(t)$, $u_2(t)$ can be considered as controls. Economic reasons provide restrictions on values of the controls:

$$u_1(t) \in U_1, \quad u_2(t) \in U_2,$$

the symbols U_1, U_2 denote compact sets.

We assume that the admissible controls $u_1(t)$, $u_2(t)$ are elements of the set

$$\mathbf{U_T} = \{\forall u(\cdot) = (u_1(\cdot), u_2(\cdot)) : [0,T] \mapsto U_1 \times U_2 \text{ are measurable}\}$$

We define coefficients in $G(p,q)$ via applying the least-squares method to the relations

$$h^*(t_i) = G(p^*(t_i), q^*(t_i)), \quad i = 0,1,\ldots,N.$$

The problem of reconstruction of controls $u_1(t), u_2(t)$ generating motions $p(t), q(t)$ close to the statistic data is considered as the optimal control problem. The controlling macroeconomic system is aiming to minimize the pay-off functional

$$I(p(\cdot), q(\cdot), u_1(\cdot), u_2(\cdot)) =$$
$$= \int_0^T \left[(p^*(t) - p(t))^2 + (q^*(t) - q(t))^2 + \varepsilon \frac{(u_1(t))^2 + (u_2(t))^2}{2} \right] dt$$

on the set $\mathbf{U_T}$. Here $p^*(t)$, $q^*(t)$ are linear interpolations for the statistic data, ε is a parameter of regularization.

The numerical method [5] is used to solve the optimal control problem [6]. The obtained constructions of the grid optimal synthesis are applied to get scenarios of a short-term prediction in order to achieve optimal decision making for two participants of the market p and q in an non-antagonistic game, where

$$G(p,q) \to \max_p, \quad G(p,q) \to \min_q.$$

See also ([7];[8])) to compare different approaches to the problems.

References

[1] Al'brekht E.G. ‘‘Methods of construction and identification of mathematical models for macroeconomic processes", electronic journal ‘‘Investigated in Russia" (http:://zhurnal.ape.relarn.ru/articles/2002/005.pdf) (in Russian).

[2] Subbotina N.N., Tokmantsev T.B. ‘‘A Numerical Method for the Minimax Solution of the Bellman Equation in the Cauchy Problem with Additional Restrictions", Proceedings of the Steklov Institute of Mathematics, Supp l, P. S222-S229, 2006.

[3] N.N. Subbotina, T.B. Tokmantsev ‘‘The Method of Characteristics to Researches on a Macroeconomic Model", Abstracts of the III International Conference "Game Theory and Management" (GMT2009), Graduate School of Management, St. Petersburg University, International Society of Dynamic Games, St.-Petersburg, Russia, pp.242-244, 2009.
[4] Tarasyev, A. M. Watanabe, C. and B.Zhu Optimal feedbacks in techno-economic dynamics. Int. J. Technol. Managem., (23(7-8), 691–717.
[5] Petrosyan, L. A. Cooperative Stochastic Games. Ann. Int. Soc. Dynamic Games, In: Adv.in Dynamic Games, Appl. Econ., Manag. Sci., Engin., and Environ. Manag., (Haurie, A. Muto, S. Petrosyan, L. A. and T. E. S. Raghavan, eds.), Birkhauser: Boston.
[61] Petrosyan, L. A. Cooperative Stochastic Games. Ann. Int. Soc. Dynamic Games, In: Adv.in Dynamic Games, Appl. Econ., Manag. Sci., Engin., and Environ. Manag., (Haurie, A. Muto, S. Petrosyan, L. A. and T. E. S. Raghavan, eds.), Birkhäuser: Boston.

ВЕСТНИК САНКТ–ПЕТЕРБУРГСКОГО УНИВЕРСИТЕТА

Серия 8
МЕНЕДЖМЕНТ

Ответственный редактор серии:
Ю.Е. Благов

Издательство Высшей школы менеджмента Санкт-Петербургского университета

A Data Transmission Game in OFDM Wireless Networks Taking into Account Power Cost

Anton Toritsyn[1], **Rimma Salakhieva**[2] and **Andrey Garnaev**[3]

[1,2,3]*Saint Petersburg State University, Russia*
[1]*foraldo@gmail.com*
[3]*garnaev@yahoo.com*

Keywords: *Wireless networks, Nash Equilibrium, Water-filling, Power cost.*

Abstract: *The goal of this work is to extend the model from Altman E., Avrachenkov K., Garnaev A. "Closed form solutions for water-filling problems in optimization and game frameworks" for the case with different fading channel gains and also taking into account cost of power unit which can produce essential impact on behaviour of users. The equilibrium strategies for the extended model are found in closed form.*

Introduction

In wireless networks and DSL access networks the total available power for signal transmission has to be distributed among several resources. This spectrum of problems can be considered in game-theoretical multiusers scenario which leads to ‘‘Water Filling Game" or ‘‘Gaussian Interference Game"([4], [5], [6], [7], [2], [3]), where each user perceives the signals of the other users as interference and maximizes a concave function of the noise to interference ratio. A natural approach in the non-cooperative setting is the application of the Iterative Water Filling Algorithm (IWFA) [8]. In [2], [3] a closed form approach to Nash equilibrium was developed for symmetric water filling game. The goal of this work is to extend this approach for scenario with different fading channel gains and also taking into account cost of power unit which can produce essential impact on behaviour of users.

Two Players Game

We consider game-theoretical formulation of the situation where two users (transmitters) transmit signals through n sub-carriers taking into account tariff assigned by provider. A strategy of user j ($j=1,2$) is vector $T^j=(T_1^j,\dots,T_n^j)$, where $T_i^j\ge 0$ and $\sum_{i=1}^n T_i^j \le \bar{T}^j$, where $\bar{T}^j>0$ is the total signal which user j has to transmit. The payoff to users are quality of service (in our case it is Shannon capacities) minus transmission expenses. Thus, the payoffs are given as follows:

$$v^j(T^1,T^2)=\sum_{i=1}^{n}\ln\left(1+\frac{T_i^1}{g_iT_i^{3-j}+N_i^0}\right)-C\sum_{i=1}^{n}T_i^{3-j}, j\in[1,2],$$

where $N_i^0>0$ is the uncontrolled noise, g_i is fading sub-carrier gains for for sub-carrier i. C is a cost per transmitted power unit.

Threorem 1 *Let* $g_i<1$. *The equilibrium strategies* $(T^1,T^2)=(T^1(\omega^1,\omega^2),T^2(\omega^1,\omega^2))$ *as functions on Lagrange multipliers have to have the following form:*

(a) Let $\omega^1\le\omega^2$. *Then*

$$\begin{aligned}
T_i^1(\omega^1,\omega^2)= &\quad \left[\frac{1}{\omega^1+C}-N_i^0\right]_+-\frac{g_i}{1-g_i^2}\left[\left(\frac{1}{\omega^2+C}-N_i^0\right)-g_i\left(\frac{1}{\omega^1+C}-N_i^0\right)\right]_+\\
T_i^2(\omega^1,\omega^2)= &\quad \frac{1}{1-g_i^2}\left[\left(\frac{1}{\omega^2+C}-N_i^0\right)-g_i\left(\frac{1}{\omega^1+C}-N_i^0\right)\right]_+
\end{aligned}$$

(b) Let $\omega^1>\omega^2$. *Then*

$$\begin{aligned}
T_i^1(\omega^1,\omega^2)= &\quad \frac{1}{1-g_i^2}\left[\left(\frac{1}{\omega^1+C}-N_i^0\right)-g_i\left(\frac{1}{\omega^2+C}-N_i^0\right)\right]_+\\
T_i^2(\omega^1,\omega^2)= &\quad \left[\frac{1}{\omega^2+C}-N_i^0\right]_+-\frac{g_i}{1-g_i^2}\left[\left(\frac{1}{\omega^1+C}-N_i^0\right)-g_i\left(\frac{1}{\omega^2+C}-N_i^0\right)\right]_+
\end{aligned}$$

Let $H(\omega^1,\omega^2)=H^1(\omega^1,\omega^2)+H^2(\omega^1,\omega^2), \text{where} H^j(\omega^1,\omega^2)=\sum_{i=1}^{n}T_i^j(\omega^1,\omega^2)$.

Threorem 2 *Let* $g_i<1$ *for any* i. *If* $H(0,0)\le\overline{T}^1+\overline{T}^2$ *then the game has unique Nash equilibrium* (T^1,T^2), *where*

(a) If $H^1(0,0)\le\overline{T}^1$ *and* $H^2(0,0)\le\overline{T}^2$ *then* (T^1,T^2) $=(T^1(0,0),T^2(0,0)$.

(b) If $H^1(0,0)>\overline{T}^1$ *and* $H^2(0,0)<\overline{T}^2$ *then* (T^1,T^2) $=(T^1(\omega_{10}^{1*},0),T^2(\omega_{10}^{1*},0))$, *where* ω_{10}^{1*} *is unique solution of equation* $H^1(\omega_{10}^{1*},0)=\overline{T}^1$.

(c) If $H^1(0,0)<\overline{T}^1$ *and* $H^2(0,0)>\overline{T}^2$ *then* (T^1,T^2) $=(T^1(0,\omega_{01}^{2*}),T^2(0,\omega_{01}^{2*}))$, *where* ω_{01}^{2*} *is unique solution of equation* $H^2(0,\omega_{01}^{2*})=\overline{T}^2$.

Threorem 3 *Let* $g_i<1$ *for any* i. *Let also* $\overline{T}^2>\overline{T}^1$. *If* $H(0,0)>\overline{T}^1+\overline{T}^2$ *then the game has unique Nash equilibrium* (T^1,T^2) *and it is given as follows:*

(a) If $H^2(\omega_{10}^{1*},0)\le\overline{T}^2$ *then* $(T^1,T^2)=(T^1(\omega_{10}^{1*},0),T^2(\omega_{10}^{1*},0))$, *where where* ω_{10}^{1*} *is unique solution of equation* $H^1(\omega_{10}^{1*},0)=\overline{T}^1$.

(b) If $H^2(\omega_{10}^{1*},0)>\overline{T}^2$ *then* $(T^1,T^2)=(T^1(\omega_{11}^{1*},\omega_{11}^{2*}),T^2(\omega_{11}^{1*},\omega_{11}^{2*}))$, *where* ω_{11}^{1*} *and* ω_{11}^{2*} *is unique solution of system of equations*

$$H^1(\omega_{11}^{1*}, \omega_{11}^{2*}) = \overline{T}^1, \quad H^2(\omega_{11}^{1*}, \omega_{11}^{2*}) = \overline{T}^2.$$

References

[1] Altman E., Avrachenkov K., Garnaev A. (2007a). Jamming game in wireless networks with transmission cost. Lecture Notes in Computer Science, 4465, 1–12.

[2] Altman E., Avrachenkov K., Garnaev A. (2007b). Closed form solutions for water-filling problems in optimization and game frameworks. ACM International Conference Proceeding Series, 321, Proceedings of the 2nd international conference on Performance evaluation methodologies and tools 2007, Nantes, France.

[3] Altman E., Avrachenkov K., Garnaev A. (2010). Closed form solutions for water-filling problem in optimization and game frameworks. Telecommunication Systems Journal, (forthcomming).

[4] L. Lai and H. El Gamal (2005). The water-filling game in fading multiple access channels, submitted to IEEE Trans. Information Theory, available at http://www.ece.osu.edu/helgamal/.

[5] O. Popescu and C. Rose (2003). Water filling may not good neighbors make in Proceedings of GLOBECOM 2003, 3, 1766–1770.

[6] D.C. Popescu, O. Popescu and C. Rose (2004). Interference avoidance versus iterative water filling in multiaccess vector channels, in Proceedings of IEEE VTC, 3, 2058–2062.

[7] W. Yu (2002). Competition and cooperation in multi-user communication environements. PhD Thesis, Stanford University.

[8] W. Yu, G. Ginis and J.M. Cioffi (2002). Distributed multiuser power control for digital subscriber lines. IEEE JSAC, 20, 1105–1115.

ВЕСТНИК САНКТ–ПЕТЕРБУРГСКОГО УНИВЕРСИТЕТА

Серия 10

ПРИКЛАДНАЯ МАТЕМАТИКА, ИНФОРМАТИКА, ПРОЦЕССЫ УПРАВЛЕНИЯ

Ответственный редактор серии:

Л.А. Петросян

Зам. ответственного редактора:

Д.А. Овсянников

С.В. Чистяков

Издательство Санкт-Петербургского университета

Recurrent Infection and Externalities in Treatment

Flavio Toxvaerd

University of Cambridge,
United Kingdom
fmot2@cam.ac.uk

Keywords: *Economic epidemiology, Susceptible-infected-susceptible models, Treatment, Externalities and complementarities, Differential games.*

Abstract: *In a basic model of organizational systems control theory a system is considered where a sole principal controls several agents. In a wide range of practical situations the agents try to build some network structure by establishing bonds to each other. The principal's role (the control problem) is to force formation of certain bonds between agents. Given the control action the agents play a network formation game (NFG). Control theory imposes strong requirements on the game-theoretic solution concepts employed. Traditional solution concepts, like Nash equilibrium, when applied to NFGs, fail to satisfy these requirements. Thus, several special strategic solution concepts were developed in the literature for NFGs. In the report these concepts are compared and aligned into the line from the weakest to the strongest. A general setting is considered of incentive problem where the principal can immediately set the bonus to the agents for the formation of specific networks. This problem is solved in complete information framework for several NGG solution concepts.*

This paper studies a model in which a population of fully rational and forward-looking individuals is exposed to a welfare reducing infectious disease. Individuals non-cooperatively choose whether to undergo privately costly treatment which, if successful, restores the individual's susceptibility. It is shown that the game has strategic complementarities, in the sense that the benefits to treatment for a given individual are increasing in the treatment of other individuals. For extreme levels of disease prevalence, equilibrium play is uniquely determined and socially optimal. For intermediate levels of disease prevalence, multiple perfect foresight equilibrium paths coexist and can lead to different steady states. Furthermore, equilibrium play may be socially suboptimal.

JEL Classification: C73, I18.

D.W.K. Yeung's Condition for the Discrete-time Government Debt Stabilization Game

Anna Tur

St. Petersburg State University
Faculty of Applied Mathematics and Control Processes
Universitetckiy pr. 35, St. Petersburg, 198504, Russia
Fax: +7 (812) 428 71 59
http://www.apmath.spbu.ru
tur.anna.v@gmail.com

Keywords: *Discrete-time Game, D.W.K. Yeung's Condition, Government Debt Stabilization Game*

Abstract: *An irrational-behavior-proofness (I-B-P) condition in cooperative differential games under which even if irrational behaviors appear later in the game the concerned player would still be performing better under the cooperative scheme is proposed by D.W.K. Yeung (2006). In this paper we study this condition for Pareto solution in linear-quadratic discrete-time dynamic games. The discrete-time government debt stabilization game is considered as an example.*

We consider the class of n-person discrete-time dynamic games which are described by the state equation

$$x(k+1) = A(k)x(k) + \sum_{i=1}^{n} B_i(k)u_i(k), \tag{1}$$

$k \geq k_0$, $k_0 \in K_+$, $x(k_0) = x_0$.

Here, x is the m-dimensional state of the system, u_i is a r-dimensional (control) variable player i can manipulate, $x(k_0) = x_0$ is the arbitrarily chosen initial state of the system, K_+ - is a set of non-negative integer numbers. $A(k), B_i(k) \in Z(K_+)$ are matrices of appropriate dimensions, where $Z(K_+)$ – is a set of real bounded matrices. Let $N = \{1, \ldots, n\}$. The performance criterion player $i \in N$ aims to maximize is

$$J_i = \sum_{k=k_0}^{\infty} (x^T(k)P_i(k)x(k) + u_i^T(k)R_i(k)u_i(k)), \tag{2}$$

where $P_i(k), R_i(k) \in Z(K_+)$ $P_i(k) = P_i^T(k)$, $R_i(k) = R_i^T(k)$ $\forall i = 1, \ldots, n$.

We will assume that the players use feedback strategies, $u_i(k,x) = M_i(k)x(k)$, to control the system.

Definition 1. A set of feedback strategies

$$\{u_i(k,x) = M_i(k)x(k), \quad i = 1,\dots,n\} \tag{3}$$

is called permissible if the following conditions are satisfied.

1) $M_i(k) \in Z(\mathsf{K}_+) \quad \forall i = 1,\dots,n.$

2) The resulting system described by

$$x(k+1) = (A(k) + \sum_{i=1}^{n} B_i(k)M_i(k))x(k) \tag{4}$$

is uniformly asymptotically stable (when $k \to \infty$).

In this paper we concretize the D. W. K. Yeung's condition for Pareto solution in proposed class of games.

As an example we consider the discrete-time variant of the government debt stabilization game proposed in (van Aarle, Bovenberg and Raithe, 1995).

References

[1] Aarle, B. van, Bovenberg, L. and Raith, M. (1995). Monetary and fiscal policy interaction and government debt stabilization. Journal of Economics, 62(2), 111-140.

[2] Markovkin, M. V. (2006). D. W. K. Yeung's Condition for Linear Quadratic Differential Games. In: Dynamic Games and Their Applications (L. A. Petrosyan & A. Y. Garnaev , eds.), St Petersburg State University, St Petersburg, 207–216.

[3] Petrosyan, L. A. and N. N. Danilov (1982). Cooperative differential games and their applications. (Izd. Tomskogo University, Tomsk).

[4] Yeung, D. W. K. (2006). An Irrational-Behavior-Proof Condition in Cooperative Differential Games. IGTR, 8(1), 739-744.

[5] Yeung, D. W. K. and L. A. Petrosyan (2004) . Subgame consistent cooperative solutions in stochastic differential games. Journal of Optimization Theory and Applications, 120(3), 651–666.

Strict Proportional Power and Fair Voiting Rules in Committee

František Turnovec

Charles University in Prague,
Institute of Economic Studies[2]
Czech Republic
frantisek.turnovec@tiscali.cz

Keywords: *Simple weighted committee, Fairness, Optimal quota, Strict proportional power, Voting and power indices*

Abstract: *In simple weighted committees with finite number n of members, fixed weights and changing quota, there exists a finite number r of different quota intervals of stable power* $(\mathrm{r} \leq (2^{\mathrm{n}})\text{-}1)$ *generating finite number of power indices vectors. If voting power is equal to blocking power, then number of different power indices vectors corresponding to majority quotas is equal to at most int*$(\mathrm{r}/2)+1$. *If the fair distribution of voting weights is defined, then fair distribution of voting power means to find a quota that minimizes distance between relative voting weights and relative voting power (optimal quota). Index of fairness is introduced as a function of quota. The problem of optimal quota has an exact solution via finite number of majority marginal quotas.*

Simple weighted committee is a pair $[\mathrm{N}, \mathrm{w}]$, where N be a finite set of n committee members $i = 1,2,...,n$, and $\mathrm{w} = (\mathrm{w}_1, \mathrm{w}_2, ..., \mathrm{w}_\mathrm{n})$ be a nonnegative vector of committee members' voting weights (e.g. votes or shares). By 2N we denote power set of N (set of all subsets of N). By voting configuration we mean an element $S \in 2^N$, subset of committee members voting uniformly (YES or NO), and $w(S) = \sum_{i \in S} w_i$ denotes voting weight of configuration S. Voting rule is defined by quota q, satisfying $0 < q \leq w(N)$, where q represents minimal total weight necessary to approve the proposal. Triple $[N, q, \mathbf{w}]$ we call a simple quota weighted committee. Voting configuration S in committee $[N, q, \mathbf{w}]$ is called a winning one if $w(S) \geq q$ and a losing one in the opposite case. Winning voting configuration S is called critical if there exists

[2] Contact address: Frantisek Turnovec, IES Faculty of Social Sciences, Charles University in Prague, Opletalova 26, 110 00 Prague 1, Czech Republic. e-mail: turnovec@fsv.cuni.cz .

at least one member $k \in S$ such that $\mathrm{w(S \backslash k) < q}$ (we say that k is critical in S). Winning voting configuration S is called minimal if any of its members is critical in S.

A priori voting power analysis seeks an answer to the following question: Given a simple quota weighted committee $[N, q, \mathbf{w}]$, what is an influence of its members over the outcome of voting? Absolute voting power of a member i is defined as a probability $\Pi_i[N, q, \mathbf{w}]$ that i will be decisive in the sense that such situation appears in which she would be able to decide the outcome of voting by her vote (Nurmi (1997)), and a relative voting power as

$$\pi_i[N, q, w] = \frac{\Pi_i[N, q, w]}{\sum_{k \in N} \Pi_k[N, q, w]}$$

Two most frequently used measures of a priori voting power are Shapley-Shubik power index (based on concept of pivot and Penrose-Banzhaf power index (based on concept of swing)

Concept of fairness is being discussed related to distribution of voting power among different actors of voting. This problem was clearly formulated by Nurmi (1982): "If one aims at designing collective decision making bodies which are democratic in the sense of reflecting the popular support in terms of the voting power, we need indices of the latter which enable us to calculate for any given distribution of support and for any decision rule the distribution of seats that is 'just'. Alternatively, we may want to design decision rules that – given the distribution of seats and support – lead to a distribution of voting power which is identical with the distribution of support."

Voting power is not directly observable: as a proxy for it voting weights are used (number of seats, number of votes, shares of population, square roots of shares of population, membership fees to some multilateral organization etc.). Therefore, fairness is usually defined in terms of voting weights (e.g. voting weights proportional to results of election).

Assuming, that a principle of fairness is selected for a distribution of voting weights, we are addressing the question how to achieve equality of voting power (at least approximately) to fair voting weights. The concepts of strict proportional power and randomized decision rule introduced by Holler (1985) and analyzed in Berg and Holler (1986), of optimal quota of Słomczyński and Życzkowski (2007), and of intervals of stable power (Turnovec (2008)) are used to find, given voting weights, a voting rule minimizing a distance between actors' voting weights and their voting power.

In the first section of the paper basic definitions are introduced and the applied power indices methodology shortly resumed. The second section introduces concepts of quota intervals of stable power and optimal quota. It is shown that in a simple weighted committees with finite number n of members, fixed weights and changing quota, there exists a finite number r of different quota intervals of stable power ($r \leq 2^N - 1$) generating finite number of power indices vectors. If voting power is equal to blocking power, then number of different power indices vectors corresponding to majority quotas is equal to at most int $(r/2) + 1$. If the fair distribution of voting weights is defined, then fair distribution of voting power means to find a quota that minimizes distance between relative voting weights and relative voting power (optimal quota). Index of fairness is introduced as a function of quota. The problem of optimal quota has an exact solution via finite number of majority marginal quotas. While the framework of analysis of fairness is usually restricted to Penrose-Banzhaf concept of power, we are treating it in a more general setting and our results are relevant for any power index based on pivots or swings and for any concept of fairness.

Acknowledgements

This research was supported by the Grant Agency of the Czech Republic, project No. 402/09/1066 "Political economy of voting behavior, rational voters' theory and models of strategic voting" and by the Max Planck Institute of Economics in Jena. The author would like to thank Manfred J. Holler and an anonymous referee for valuable comments to earlier version of the paper.

References

[1]. Berg, S. and M.J. Holler (1986), Randomized Decision Rules in Voting Games: a Model for Strict Proportional Power, Quality and Quantity, 20, 419-429.

[2]. Holler M.J. (1985), Strict Proportional Power in Voting Bodies, Theory and Decision, 19, 249-258.

[3]. Nurmi H. (1982), The Problem of the Right Distribution of Voting Power. In: Holler M.J. (ed.), Power, Voting and Voting Power, Physica Verlag, Würzburg, Wien, 203-212.

[4]. Nurmi, H. (1997), On Power Indices and Minimal Winning Coalitions. Control and Cybernetics, 26, 609-611.

[5]. Słomczyński W. and K. Życzkowski (2007), From a Toy Model to Double Square Root System, Homo Oeconomicus, 24, No. 3-4, 381-400.

[6]. Turnovec F. (2008b), Efficiency of Fairness in Voting Systems, Homo Oeconomicus, 25, No. 3-4, 335-348.

Optimal Tax Enforcement with Corruptible Auditors

Anton Urazov[1] and **Alexander Vasin**[2]

[1,2]*Lomonosov Moscow state University*
Russia
[1]*vasin@cs.msu.su*
[2]*anton@urazov.me*

Keywords: *Tax evasion, Corruption, Game-theoretic modelling*

Abstract: *We consider several problems of optimal organization for revenue collecting inspections. We study whether it is possible to organize an effective control over 100000 agents and suppress corruption in case when very few reliable persons are available for this purpose. We determine the optimal strategy including selection of agents for the primary audit and revisions, salaries at different levels. We show that under general assumptions 5-level inspection and one honest person are enough for efficient tax enforcement. .*

The state inspections play an important role in the modern economy. The tax inspections and the customs control the payment's values and check examptions from payments for different economic agents. The agency should prevent tax or customs evasion but not interfere with the agents eligabale for examption from the payment. The efficiency of an inspection should be measured by the social welfare increase proceding from its activity.

For many countries in transition, in particular for Russia, corruption is an important problem in tax and customs inspections' organization. Bribery is one form of corruption that is the most difficult to reveal. There exists a wide literature that discusses problems of optimal inspection organization (in particular for tax inspection) and the problem of corruption.

One branch of the literature (Chander, Wilde, 1992, Acemoglu, Verdier, 2000) considers civil society institutes (independent mass media and so on) as a main mechanism of corruption revealing. Another direction (Vasin, Panova, 2000) studies internal control strategies for suppression of corruption. The latter paper assumes that there is a possibility to hire sufficient number of honest collabolators for reviewing primary audits, but actually the controlling center typically has very few reliable persons, and their time is a very expensive resource.

Then one possibility is to form a controlling hierarchy. Consider a country where a benevolent leader aims to organize an efficient tax audit. There are N firms. Each gets high or low income with probabilities h and $1-h$ respectively. The additional tax from the high income is T, and the penalty for evasion is F. For the inspection, the leader can use a small number M of reliable associates and also employ any number of rational inspectors who maximize their expected incomes with account of possble salaries, bribes and penalties. Salary s_M permits to employ a sufficient number of such inspectors, and $\tilde{c}$ is the cost of one audit by an associate. Consider a strategy of the revenue service organization. It includes probability p_1 of primary audit for any low-income declaration. In order to prevent bribing of a primary auditor, any report confirming low income is under reviewing with probability p_2. And so on, any i-level audit confirming the low income is under reviewing ($i+1$-level audit) with probability p_{i+1} until the upper level k where associates work. A salary of an i - level inspector is $s_i \geq s_M$, $i = 1,...,k-1$. Each revealed inspector who has not reported tax evasion is fired and gets after that alternative salary s_{alt}. This value is uncertain: we assume that $s_{alt} \in (s_M - \Delta, s_M)$. Thus, a government strategy includes the number k of audit levels, auditing probabilities $p_1,...,p_k$ and salaries $s_1,...,s_{k-1}$.

A formal problem is to find the optimal strategy that provides honest behaviour of all agents and maximises net tax revenue (NTR) under this condition. Note that, for risk-neutral inspector, firing is equivalent to monitary fine $\tilde{F} = (s - s_{alt})\alpha$, $\alpha = \delta/(1-\delta)$, where δ is a discount coefficient. Let $d_i = s_i - s_M$ denote the increment of the salary at level i above the maximum alternative salary.

Proposition 1. *Assume that auditors at level i check honestly. Then mutualy beneficial collusion between $i-2$ level inspector and his auditor is impossible if and only if* $p_i \geq \dfrac{d_{i-2} + \Delta}{d_{i-2} + d_{i-1} + \Delta} = \hat{p}_i(\vec{d})$ *for* $i = 3,...,k$. *Tax evasion is unprofitable if and only if* $p_1 \geq \dfrac{T}{F} = \hat{p}_1(\vec{d})$ *and collusion between tax payer and his auditor is impossible if and only if* $p_2 \geq \dfrac{F}{F + d_1\alpha} = \hat{p}_2(\vec{d})$.

Consider k-stage game corresponding to the random interaction of inspectors in the hierarchy.

Proposition 2. *The subgame perfect equilibrium corresponding to the honest behaviour in the interaction of inspectors and taxpayers exists if and only if the auditing probabilities meet Proposition 1.*

Under fixed $k, \vec{d}$ *the NTR reaches its maximum at* $p_i = \hat{p}_i(\vec{d})$, $i = 1,...,k$ *and is equal to*

$$R(k,\vec{d}) = hT - \hat{p}_1(\vec{d})(1-h)(s_M + d_1 + \hat{p}_2(\vec{d})(s_M + d_2 + \hat{p}_3(\vec{d}) \times (...\hat{p}_{k-1}(\vec{d})(s_M + d_{k-1} + \hat{p}_k(\vec{d})\tilde{c})...)$$

The expected number of audits by associates is $N(1-h)\prod_{i=1}^{k}\hat{p}_i(\vec{d})$.

Proposition 3. *For* $\Delta = 0$ *the lower estimate of the optimal revenue is* $\bar{R} = N(hT - (1-h)\frac{T}{F}2s_M)$. *For* $d_i = \varepsilon^{k-i}, i = 1,...,k-1$ $R(k,\vec{d})$ *tends to* $\bar{R}$ *as* $\varepsilon \to 0, k \to \infty$. *The optimal revenue is equal to* $\bar{R}$ *if* $\min(d_1 + \frac{F}{F + d_1\alpha}s_M)$ *reaches its minimum under* $d_1 = 0$.

For $\Delta > 0$ and different k we solved the problem of NTR maximization by the dynamic programming method. We also determined the necessary number of associates to realize the optimal strategy.

Consider the following example. The additional tax from the high income is $T = 300000$ rubles and the penalty for evasion is $F = 8T$. The number of tax payers is $N = 100000$, the probability to get high income is $h = 0{,}5$. Associates get $\tilde{c} = 3000000$ rubles per one check. Each auditor can make 60 inspections or revisions per year. Let a discount coefficient equal $\delta = 0{,}1$.

Under these conditions, the gross tax revenue should be $NhT = 15$ bn. rubles. The optimal NTR is $R^* = NhT - E^*$, where E^* is total audit costs under the optimal strategy $\vec{d}^*$. The table below shows the optimal NTR, the ratio Z of the cost E^* to the lower estimate of the costs $\underline{C} = (1-h)\frac{T}{F}s_M N$, the necessary number M of honest associates, optimal salaries (in thousands rubles per year) and auditing probabilities for different k, Δ, s_M.

k	4				5			
s_M	225	270	405	450	225	270	405	450
Δ	90	180	90	180	90	180	90	180
NTR	14 615 670	14 415 960	14 576 700	14 376 600	14 781 240	14 659 590	14 739 900	14 617 500
M	1	2	1	2	1	1	1	1
Z	16,40	20,77	10,03	13,30	9,33	12,10	6,17	8,16
s_1	225	270	405	450	225	270	405	450
s_2	1287	1908	1467	2106	999	1548	1197	1746
s_3	13 473	16 542	13 653	16 794	225	270	405	450
s_4	180 000	180 000	180 000	180 000	4 167	5 778	4 347	5 958
s_5					180 000	180 000	180 000	180 000
p_1	0,125	0,125	0,125	0,125	0,125	0,125	0,125	0,125
p_2	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
p_3	0,078	0,099	0,078	0,098	0,104	0,123	0,102	0,122
p_4	0,080	0,100	0,080	0,101	1,000	1,000	1,000	1,000
p_5					0,022	0,032	0,022	0,032

The results confirm possibility of efficient tax enforcement with suppression of corruption and tax evasion. Starting with 4-level inspection, the optimal auditing costs are less than 5% of the gross tax income. In order to control 100000 taxpayers it suffices to employ 220 risk-neutral inspectors and 2 associates. The average number of audits per one taxpayer is less than ¼.

Investments under Oligopolistic Competition in a Vintage Differential Game

Stefan Wrzaczek[1] and **Peter M. Kort**[2]

[1] *Vienna University of Technology, Austria*
wrzaczek@server.eos.tuwien.ac.at

[2] *University of Tilburg, Netherlands*

Keywords: *Differential game, Vintage, Time- and age-consistency*

Due to technological progress new machines are more productive. Thus it is not only important for firms to chose the optimal amount of investments, but also the optimal age of the machines. Feichtinger, Hartl, Kort and Veliov (2006a and 2006b) study this topic in case of a monopolistic firm by using distributed optimal control theory. We extend this approach to an oligopoly. The market is assumed to consist of two big firms, who are connected by the price they get for the products. The higher is the production of firm i the lower is the price of firm j and vice versa.

In order to include the vintage effect of investments we have to combine distributed optimal control theory with differential game theory (referred to as vintage differential game). As in that framework there are two independent variables (age and time) the usual concept of time consistency (as well as subgame perfectness) can be be defined for both directions. The differences between both concepts will be pointed out intuitively by simple examples. In our model we study an open-loop Nash equilibrium, for which time consistency can be shown. The proof of the time consistency can be extended to a whole class of vintage differential games. In contrast an important difference between age- and time-consistency can be derived, i.e. there are models that are time- but not age-consistent. Further we provide conditions under which a vintage differential game can never be time consistent. An intuitive reasoning of these conditions finish the theoretical part of the contribution.

We derive several properties of the optimal investment path of the model as well as comparative statics. For the expressions we are able to give interesting economic intuitions. Futher it is possible to determine the terms that arise due to including competition to the model of a monopolistic firm.

In the numerical part we analyse several different scenarios of the model. We study the effects of different parameters influencing the prices of the competing firms. Firstly we can show that the optimal investments decrease the higher the productivity of new machines is. This result can also be obtained analytically. Futher we compare the investments when the firms can anticipate the shock and when they do not (because of a lack of knowledge). The results show quite different investment patterns, as well as a possible shift in the market share (when the firms anticipate the shock differently).

References

[1] Feichtinger, G., Kort, P. M., Hartl, R. F., Veliov, V. M., 2006. Anticipation Effects of Technological Progress on Capital Accumulation: a Vintage Capital Approach. Journal of Economic Theory 126, 143-164.

[2] Feichtinger, G., Hartl, R. F., Kort, P. M., Veliov, V. M., 2006. Capital accumulation under technological progress and learning: A vintage capital approach. European Journal of Operational Research 172 (1), 293-310.

Game Theory Models for Supply Chains Optimization

Victor Zakharov[1] and **Mansur Gasratov**[2]

[1]*Saint Petersburg Universitet*
Russia
[1]*mcvictor@mail.ru*
[2]*gasratovmans@mail.ru*

Keywords: *Game Theory, Supply Chains*

Abstract: *In this paper game theoretic mathematical models of logistic systems are treated. We consider the network of nodes connected by the logistic canals. In each node production and commercial structures (distributors) with their warehouses are allocated, and a demand for substituted goods has deterministic nature. The demand for a good is influenced by the quantity and price that is we consider quantitative and price competition among distributors. For the modelling of management systems each production and commercial structure applies the relaxation method of inventory regulation with admission of the deficit.*

Distributors are considered as players in non cooperative game. The logistic networking process are divided into two levels: at the first level the management systems of logistic processes are optimized by choosing internal strategies $\boldsymbol{T^l} = \{T_{k_l}\}_{k_l=1}^{p_l} \in R^{p_l}$ and $\boldsymbol{S^l} = \{S_i^l\}_{i=1}^{n} \in R^n$ (interior problem), where T_{k_l} is the period of a cycle over routing path which have number k_l, p_l is a quantity of the routing paths for the distributor with number l in the network $(p_l \le n)$, $l = 1,...,N$, S_i^l is maximal inventory level of distributor l in the node number i, $i = 1,...,n$; at the second level (exterior problem) a non-cooperative game for the price and quantitative type of competition is considered and solved. In this problem we use Nash equilibrium in pure (external) strategies as a solution. In the case of quantitative (model of Cournot) competition vector of amount of supplied goods to warehouses located in all nodes $\boldsymbol{Q^l} = \{Q_i^l\}_{i=1}^{n} \in R^n$ is an external strategy of player l. In case of price competition (modified model of Bertrand) we assume vector $\boldsymbol{p^l} = \{p_i^l\}_{i=1}^{n} \in R^n$ as an external strategy of player l, where p_i^l is the selling price for good supplied by distributor with number l at the node with number i. We use marginal revenue for distributors as payoff functions to the players in the game.

For the both cases of competition sufficient conditions for existence of Nash equilibrium in pure strategies are formulated. In addition, formulas for calculating optimal strategies in internal problem are provided.

Theorem 1. *Suppose in Cournot model the following conditions hold for* $i = 1,\dots,N$ *:*

1) Reveres demand function $p\left(Q_i,\mathbf{Q_{-i}}\right)$ *is twice differentiable, deceasing and concave with respect to* $Q_i \in \Omega_i^{(1)}$ *for any fixed set of* $\mathbf{Q_{-i}} \in \Omega^{(1)} / \Omega_i^{(1)}$ *;*

2) Function $\dfrac{Q_i}{\sqrt{b_i\left(Q_i,\mathbf{Q_{-i}}\right)}}$ *is continuous with respect to* $\Omega_i^{(1)}$ *and convex with respect to* $\mathbf{Q_{-i}}$

or the following inequality takes place (if $b_i\left(Q_i,\mathbf{Q_{-i}}\right)$ *- twice differentiable);*

$$3Q_i\left(\frac{\partial b_i(Q_i,Q_{-i})}{\partial Q_i}\right)^2 \geq 4b_i(Q_i,Q_{-i})\frac{\partial b_i(Q_i,Q_{-i})}{\partial Q_i} + 2Q_i b_i(Q_i,Q_{-i})\frac{\partial^2 b_i(Q_i,Q_{-i})}{\partial Q_i^{\,2}}$$

3) There exists $\tilde{Q}_i \in \left(a_i^{(1)},b_i^{(1)}\right)$ *:*

$$p(Q_i,Q_{-i}) < \sqrt{\frac{2K_i g_i h_i}{g_i + h_i}} \cdot \frac{b_i(Q_i,Q_{-i}) - 1/2Q_i\dfrac{\partial b_i(Q_i,Q_{-i})}{\partial S_i}}{b_i^{3/2}(Q_i,Q_{-i})} + c_i$$

for $Q_i \geq \tilde{Q}_i$.

Then there exists Nash equilibrium in pure strategies $\left(Q_1^*,Q_2^*,\dots,Q_N^*\right)$, $Q_i^* \in \left[a_i^{(1)},\tilde{Q}_i\right)$, $i = 1,\dots,N$ *and this strategies form appears a solution of the following system*

$$\frac{\partial p(Q_i,Q_{-i})}{\partial Q_i}Q_i + p(Q_i,Q_{-i}) - \sqrt{\frac{2K_i g_i h_i}{g_i + h_i}} \cdot \frac{b_i(Q_i,Q_{-i}) - 1/2Q_i\dfrac{\partial b_i(Q_i,Q_{-i})}{\partial Q_i}}{b_i^{3/2}(Q_i,Q_{-i})} - c_i = 0,\ i = 1,\dots,N.$$

Theorem 2. *Suppose in modified model of Bertrand the following conditions hold for* $i = 1,\dots,N$ *:*

1) Function $D\left(p_i,\mathbf{p_{-i}}\right)$ *is continuous, decreasing, differentiable and convex with respect to*

$p_i \in \Omega_i^{(1)}$ *for each fixed* $\mathbf{p_{-i}} \in \Omega^{(1)} / \Omega_i^{(1)}$ *;*

2) Functiion $D_i\left(p_i,\mathbf{p_{-i}}\right)\Big/\sqrt{b_i\left(p_i,\mathbf{p_{-i}}\right)}$ *is continuous convex with respect to* $p_i \in \Omega_i^{(1)}$ *for fixed admissible* $\mathbf{p_{-i}}$ *;*

3) Function $p_i D_i\left(p_i,\mathbf{p_{-i}}\right)$ *is concave with respect to* $p_i \in \Omega_i^{(1)}$ *for fixed* $\mathbf{p_{-i}} \in \Omega^{(1)} / \Omega_i^{(1)}$.

Then in the game (4) there exists Nash equilibrium in pure strategies which is a solution of the system

$$\frac{\partial D_i(p_i,p_{-i})}{\partial p_i} b_i(p_i,p_{-i}) + D_i(p_i,p_{-i}) =$$

$$= \xi_i \frac{2\frac{\partial D_i(p_i,p_{-i})}{\partial p_i} b_i(p_i,p_{-i}) - D_i(p_i,p_{-i})\frac{\partial b_i(p_i,p_{-i})}{\partial p_i}}{2b_i^{3/2}(p_i,p_{-i})} + \frac{\partial D_i(p_i,p_{-i})}{\partial p_i} c_i,\ \ i = 1,\ldots,N.$$

If there exists $\tilde{p}_i \in \left(a_i^{(1)}, b_i^{(1)}\right)$ *so that*

$$D_i(p_i,p_{-i}) < \xi_i \frac{2\frac{\partial D_i(p_i,p_{-i})}{\partial p_i} b_i(p_i,p_{-i}) - D_i(p_i,p_{-i})\frac{\partial b_i(p_i,p_{-i})}{\partial p_i}}{2b_i^{3/2}(p_i,p_{-i})} + \frac{\partial D_i(p_i,p_{-i})}{\partial p_i} c_i$$

$p_i \geq \tilde{p}_i$ *, then* $p_i^* \in \left[a_i^{(1)}, \tilde{p}_i\right)$.

Memento Ludi: Information Retrieval from a Game-Theoretic Perspective

Roman R. Zapatrin[1] and **Georges Parfionov**[2]

[1]*Department of Information Science, The State Russian Museum, Inzenernaya 4, 191186, St.Petersburg, Russia*
Roman.Zapatrin@gmail.com,

[2]*Department of Mathematics, SPb EF University, Griboyedova 30–32, 191023*
St.Petersburg, Russia
GogaParf@gmail.com

Keywords: *Inverse problem, Information retrieval, Users behavior*

Abstract: *We develop a macro-model of information retrieval process using Game Theory as a mathematical theory of conflicts. We represent the participants of the Information Retrieval process as a game of two abstract players. The first player is the `intellectual crowd' of users of search engines, the second is a community of information retrieval systems. In order to apply Game Theory, we treat search log data as Nash equilibrium strategies and solve the inverse problem of finding appropriate payoff functions. For that, we suggest a particular model, which we call Alpha model. Within this model, we suggest a method, called shifting, which makes it possible to partially control the behavior of massive users. This Note is addressed to researchers in both game theory (providing a new class of real life problems) and information retrieval, for whom we present new techniques to control the IR environment..*

The techniques we present are inspired by the success of macro-approach in both natural and social science. In thermodynamics, starting from a chaotic motion of billions of billions of microparticles, we arrive a simple transparent strongly predictive theory with few macro-variables, such as temperature, pressure, and so on. In models of market behavior the chaotic motion is present as well, but there are two definite parties, each consisting of a big number of individuals with common interests, whose behavior is not concorded.

From a global perspective, information retrieval looks similar: there are many individual seekers of knowledge, on one side, and a number of knowledge providers, on the other: each are both chaotic and non-concorded. There are two definite parties, whose members have similar interests, and every member of each party tends to maximally fulfill his own interests. How could a Mathematician help them? At first sight, each party could be suggested to solve a profit maximization problem. But back in

1928 it was J. von Neumann who realized this approach to be inadequate: you can not maximize the value you do not know [1]. In fact, the profit gained by each agent depends not only on its actions, but also on the activities of its counterpart, which are not known. Then the game theory was developed replacing the notion of optimality by that of acceptability. Similarly, the crucial point of information retrieval, in contrast to data retrieval, is to get some satisfaction (feeling of relevance) rather than retrieve something exact. The analogy

Data Retrieval $\rightarrow$ matching		**Optimization** $\rightarrow$ maximum
Information Retrieval $\rightarrow$ relevance	$\simeq$	**Game Theory** $\rightarrow$ equilibrium

was a starting point for us to explore applications of game theory to the problems of information retrieval.

The standard problem of game theory is seeking for reasonable (in various senses) strategies. When the rules of the game are given, there is a vast machinery, which makes it possible to calculate such strategies. In information retrieval we have two parties whose interaction is of exactly game nature, but the rules of this game are not explicitly formulated. However, we may observe the consequence of these rules as users behavior, that is, we deal with the inverse problem of game theory, studied by Dragan [2] for cooperative games. In this Note we expand it to non-cooperative case.It turns out that the solution of the inverse problem is essentially non-unique: different rules can produce the same behavior. We suggest a particular class of models, called Alpha models describing an idealized search system similar to Wolfram Alpha engine.

What can search engine managers benefit of these techniques? Game theory can work out definite recommendations how to control the interaction between the parties of the information retrieval process. This sounds unrealistic: can one control massive chaotic behavior? Thermodynamics shows us that the answer is yes. We can not control individual molecules, but in order to alter their collective behavior we are able to change macroparameters: the engine of your car reminds it to you. In our case the payoff functions of the Alpha model are just those parameters.

So far, we have described the process of Information retrieval as a non-antagonistic conflict between two parties: The User and The Provider. The mathematical model of such conflict is a bimatrix cooperative game. Starting from the assumption that de facto search log statistics is the Nash equilibrium of certain game, we provide a method of calculating the parameters of this game, thus solving the appropriate inverse problem.

A significant, somewhat counter-intuitive consequence of Nash theory is that in this class of games the equilibrium, i. e. stable, behavior of the User is completely determined only by the distribution of priorities of the Provider. From this, we infer suggestions for the provider how to affect the behavior of massive User.

References

[1] John von Neumann, Zur Theorie der Gesellschaftsspiele (Theory of parlor games), Math. Ann. 100 295 (1928)

[2] Irinel Dragan, Values, potentials and inverse problems in cooperative Game Theory, Internation J. of Pure and Applied Math. 22, 545–561

[3] Tucker, Albert, "A two-person dilemma", Stanford University Press, (1950)

[4] Owen, G. "Game Theory", Academic Press, UK, 1995.

[5] Yule, G. "The Statistical Study of Literary Vocabulary", Cambridge University Press, (1944)

МАТЕМАТИЧЕСКАЯ ТЕОРИЯ ИГР И ЕЕ ПРИЛОЖЕНИЯ

Ответственный редактор:
Л.А. Петросян
Зам. ответственного редактора:
В.В. Мазалов
Ответственный секретарь:
Н.А. Зенкевич

Выходит ежеквартально с января 2009 года

Quality Choice under Competition: Game Theoretical Approach

Nikolay A. Zenkevich[1] and **Margarita A. Gladkova**[2]

Graduate School of Management, St. Petersburg State University,
Volkhovsky Per. 3 St. Petersburg, 199004, Russia
[1]*zenkevich@gsom.pu.ru*
[2]*gladkova@gsom.pu.ru*

Keywords: *Quality evaluation, Quality measurement, Consumer's taste for quality, Quality choice, Two-stage game, Nash equilibrium, Stakelberg equilibrium, Pareto-optimal solution, Optimal quality differentiation, Index of consumers' satisfaction.*

Abstract: *A game-theoretical model of quality choice under competition is suggested. It is presented as a two-stage game where production companies compete on an industrial market and consumer's taste to quality in non-uniformly distributed. The strong Nash equilibrium in the investigated game was obtained in explicit form which allowed us to evaluate prices, companies market shares and revenues in the equilibrium. A case study for Internet-trading systems was used to approve the suggested quality choice mechanism.*

In the paper a game-theoretical model of quality choice under competition is suggested. The game-theoretical model is presented as a two-stage game where production companies compete on an industrial market and consumer's taste to quality is non-uniformly distributed. From a practical perspective, the aim of the paper is to check the validity of the suggested game-theoretical model.

Thus, two firms are assumed to produce homogeneous product differentiated by quality on some industrial market. The game consists of the following two stages:

1. At the first stage companies simultaneously define quality level;

2. At the second stage they choose product prices. At this stage both simultaneous and sequential choices are analyzed.

Each consumer buys at most one unit of the good. Consumers differ in their willingness to pay for quality level s, which is described by the parameter $\theta \in [0,b]$. This parameter is called "taste for quality". The utility of a consumer with a willingness to pay for quality θ when buying a product of quality s at a price p is equal to:

$$U_\theta(p) = \begin{cases} \theta s - p, & p \le \theta s \\ 0, & p > \theta s \end{cases}.$$

The investigated industrial market is considered to be partially covered. Besides, in our model the case when taste for quality parameter is non-uniformly distributed over the interval $[0,b]$, namely the triangular distribution is investigated.

The payoff function of the firm i which produces the product of quality s_i, where $s_i \in [\underline{s}, \overline{s}]$, is the following:

$$R_i(p_1, p_2, s_1, s_2) = p_i(s_1, s_2) D_i(p_1, p_2, s_1, s_2),\ i = 1,2,$$

where $D_i(p_1, p_2, s_1, s_2)$ – the demand function for the product of quality s_i, which is specified.

The strong Nash equilibrium in the investigated game was obtained in the explicit form which allowed us to evaluate prices, companies' market shares and revenues in the equilibrium.

The paper includes a case study for Internet-trading systems which was used to approve the suggested game-theoretical approach to quality choice. Processing an empirical data which was obtained from consumer questionnaire survey let us evaluate Internet-trading systems quality and using the model we can show the ways of performance improvement to production companies.

References

[1] Aoki R., Pursa T. J. (1996). Sequential versus simultaneous choice with endogenous quality. Intenational Journal of Industrial Organization, Vol. 15, pp. 103–121.

[2] Benassi C., Chirco A., Colombo C. (2006). Vertical differentiation and distribution of income. Bulletin of Economic Research: 58(4), pp. 0307–3378.

[3] Gladkova M.A., Zenkevich N.A. (2007).Game-theoretical model "quality-price" under competition on the industry market. Vestnik of Saint Petersburg University. Management Series. Issue 4. pp. 3-31. (in Russian).

[4] Gladkova M., Zenkevich N. (2009). Quality Competition: Uniform vs. Non-uniform Consumer Distribution.Contributions to Game Theory and Management. Vol II. Collected papers presented on the Second International Conference "Game Theory and Management"/ Editors Leon A. Petrosjan, Nikolay A. Zenkevich - SPb, Graduate School of Management, SPbU, pp. 111-124.

[5] Hovanov K.N., Hovanov N.V. (1996).DSSS "ASPID-3W". Decision Support System's Shell "Analysis and Synthesis of Parameters under Information Deficiency - 3rd modification for Windows". "ASPID-3W" registered by Federal Agency for Computer Programs Coryright Protection (Russian Federation, Num. 960087).

[6] Hovanov N., Yudaeva M., Hovanov K. (2009). Multicriteria estimation of probabilities on basis of expert non-numeric, non-exact and non-complete knowledge. European Journal of Operational Research, Vol. 195, pp. 857-863.

[7] Motta M. (1993). Endogenous quality choice: price vs. Quantity competition. The journal of industrial economics, Vol. XLI, No. 2, pp. 113–131.
[8] Noh Y.-H., Moschini G. (2006). Vertical product differentiation, entry-deterrence strategies, and entry qualities. Review of Industrial Organization, Vol. 29, pp. 227–252.
[9] Petrosyan L.A., Zenkevich N.A., Semina E.A. (1998).Game theory: University textbook. Vyssh, shkola. Knizhniy dom ”Universitet”. Moscow. (in Russian).
[10] Tirole J. (1988). The theory of industrial organization. MIT Press, Cambridge, Mass.

A Game-Theoretic Model of Territorial Environmental Production

Nikolay A. Zenkevich[1] and **Nadezhda V. Kozlovskaya**[2]

[1]*St Petersburg State University*
Graduate School of Management
zenkevich@som.pu.ru

[2]*St. Petersburg University*
Faculty of Applied Mathematics and Control Processes
kknn@yandex.ru

Keywords: *Cooperative game, Shapley value.*

Abstract: *A game-theoretic model of territorial environmental production is studied. The process is modeled as cooperative differential game. The stable mechanism of distribution of the common cooperative benefit among players is proposed. We proved that the cooperative total stock of accumulated pollution is strictly less then the pollution under Nash equilibrium for the whole duration of the game. The perfect Nash equilibrium is found. We design a stable Shapley value as a cooperative solution, which is time-consistent. The Shapley value is also strategic stable and satisfies the irrational-behavior-proofness condition. The numerical example is given.*

Consider a region market with n firms which supply one product. Let I be the set of firms involved in the game: $I = \{1, 2, \ldots, n\}$. Denote by $q_i = q_i(t)$ the output of firm i at the instant of time t. The price of the product $p = p(t)$ is defined as follows: $\mathrm{p(t) = a - bQ(t)}$, where $\mathrm{a > 0, b > 0}$, $Q(t) = \sum_{i=1}^{n} q_i(t)$ - the total output. The price function $p(t)$ is inverse demand function:

$$Q = Q(t) = \frac{a - p(t)}{b}.$$

The production cost of any firm equals $C_i(q_i(t)) = cq_i(t), \quad c > 0, i \in I.$

The game starts at the instant of time t_0 from the initial state s_0, where s_0 is the stock of pollution at time t_0. Let us denote by $e_i(t)$ the emission of firm i at time t. The emission of firms are linear subject to output:

$$e_i(q_i(t)) = \alpha q_i(t), \quad \alpha > 0.$$

Denote by $\overline{e}_i$ maximum permissible emission for firm i: $0 \le e_i(q_i(t)) \le \overline{e}_i$.

Denote by $s = s(t)$ the total stock of accumulated pollution by time t. The dynamics of pollution accumulation is defined by the following differential equation:

$$\dot{s}(t) = \alpha \sum_{k=1}^{n} q_i(t) - \delta s(t),$$

$$s(t_0) = s_0$$

where δ is the rate of pollution absorption, $\alpha > 0$ is the parameter. Any firm has two types of costs, which isn't directly connected with the production process: abatement costs and damage costs. The abatement costs at moment of time t equals

$$E_i(t) = \frac{\gamma}{2} e_i(t)(2\overline{e}_i - e_i(t)) = \frac{\gamma}{2} \alpha q_i (2\overline{e}_i - \alpha q_i), \ \gamma > 0, \ 0 \le e_i(t) \le \overline{e}_i.$$

Damage costs depends on the stock of pollution: $D_i(s(t)) = \pi_i s(t), \quad \pi_i > 0, \ i \in I.$

Any firm tries to maximize the the profit

$$\Pi_i(s_0, t_0; q) = \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \{ p q_i - C_i(q_i) - D_i(s) - E_i(q_i) \} dt$$

where $q = q(t) = (q_1(t), q_2(t), \ldots, q_n(t)), \quad t \ge t_0$ is trajectory of production output, $0 < \rho < 1$ is the common social discount rate.

We compute the characteristic function for all possible coalitions and apply the Shapley value to determine a fair distribution of the common cooperative benefit among players. There are three important aspects which must be taken into account when the problem of stability of long-range cooperative agreements is investigated [3]: time-consistency of the cooperative agreements, strategic stability and irrational behavior proofness. We design a stable mechanism for allocation over time of total individual benefit so that the initial agreement remains valid for the whole duration of the game. The mechanism is also strategic stable and satisfies the irrational-behavior-proofness condition. We proved that the stable Shapley value total stock of accumulated pollution is strictly less then the pollution under Nash equilibrium for the whole duration of the game.

References

[1] Petrosyan L., Kozlovskaya N. Differential coalitional environmental management game // Game theory and applications. Russia, St.Petersburg State University. Vol. 14.

[2] Petrosyan L., Zaccour G. Time-consistent Shapley value allocation of pollution cost reduction // Journal of Economic Dynamics and Control. 2003. Vol. 27. P. 381–398.

[3] Petrosyan L.A., Zenkevich N.A. Conditions for sustainable cooperation // In: Contributions to game theory and management. Edited by L.A. Petrosyan and N.A Zenkevich. P.344–354.

[4] Yeung D. W. K. An irrational - behavior - proofness condition in cooperative differential games // Intern. J. of Game Theory Rew. 2006. Vol. 8. P. 739–744.

Strong Equilibrium in Differential Games

Nikolay Zenkevich[1] and **Andrew V. Zyatchin**[2]

St.Petersburg State University,
Graduate School of Management,
Volkhovsky per.3, St.Petersburg, 199004,
Russia
[1]*zenkevich@som.pu.ru*
[2]*zyatchin.work@gmail.com*
http://www.gsom.pu.ru/en/professors/zyatchin

Keywords: *Nash equilibrium, strong equilibrium, dynamic programming*

Abstract: *In this paper two definitions of strong equilibrium are considered: strong equilibrium in broad and narrow senses. The main result of the paper is sufficient conditions for strong equilibrium existence in differential games. A special technique based on scalarization of vector criteria is used to construct strong equilibrium in broad sense. This approach is tested on example of the differential game with three players.*

Strong equilibrium definition

Consider the differential game $\Gamma(x_0, T - t_0)$ with initial state x_0 and finite duration $T - t_0$, where $t_0 \geq 0$, $T \geq t_0$ [3, 4]. Denote set of players as $N = \{1,\dots,i,\dots,n\}$.

The dynamics of the game has the following form:

$$\dot{x}(t) = f\left[t, x(t), u_1(t), \dots, u_n(t)\right], \quad x(t_0) = x_0, \tag{1}$$

where $x(t) \in R$, $u_i(t)$, $t \in [t_0, T]$ - control function of player $i \in N$, t, $u_i(t) \in U_i \subset R$, $\prod_{i=1}^{n} U_i = U_N \subset R^n$. Suppose, that for function $f\left[t, x(t), u_1(t), \dots, u_n(t)\right]$ conditions for existing and uniqueness on $[t_0, T] \times R \times U_N$ are hold.

The payoff of player $i \in N$ is defined as:

$$J_i(x_0, u_1(\cdot), u_2(\cdot), \dots u_n(\cdot)) = \int_{t_0}^{T} g_i\left[t, x(t), u_1(t), u_2(t), \dots, u_n(t)\right] dt + q_i\left[x(T)\right],$$

where $u_i(\cdot)$ is continuous function $u_i(t)$, $t \in [t_0, T]$.

Let $S \subseteq N$ is a coalition in the game $\Gamma(x_0)$. Denote strategy of the coalition S as $u_S(\cdot) = \{u_i(\cdot)\}_{i \in S}$. A strategy of additional coalition $N \setminus S$ denote as $u_{N \setminus S}(\cdot)$ or $u_{-S}(\cdot)$.

Definition. The set of strategies $u_N^*(\cdot) = (u_1^*(\cdot), u_2^*(\cdot), \ldots, u_n^*(\cdot))$ is said to constitute a strong equilibrium in broad sense for differential game $\Gamma(x_0, T - t_0)$ if $\forall M \subseteq N$, $\forall u_M(\cdot)$ the following is not hold: $\forall i \in M$

$$J_i(x_0, u_M(\cdot), u_{-M}^*(\cdot)) = \int_{t_0}^{T} g_i\left[t, x^{[M]}(t), u_M(t), u_{-M}^*(t)\right]dt + q_i\left[x^{[M]}(T)\right] \geq$$

$$\geq \int_{t_0}^{T} g_i\left[t, x^*(t), u_M^*(t), u_{-M}^*(t)\right]dt + q_i\left[x^*(T)\right] = J_i(x_0, u_M^*(t), u_{-M}^*(t))$$

and $\exists i_0 \in M$ such that:

$$J_{i_0}(x_0, u_M(\cdot), u_{-M}^*(\cdot)) = \int_{t_0}^{T} g_{i_0}\left[t, x^{[M]}(t), u_M(t), u_{-M}^*(t)\right]dt + q_{i_0}\left[x^{[M]}(T)\right] >$$

$$> \int_{t_0}^{T} g_{i_0}\left[t, x^*(t), u_M^*(t), u_{-M}^*(t)\right]dt + q_{i_0}\left[x^*(T)\right] = J_{i_0}(x_0, u_M^*(\cdot), u_{-M}^*(\cdot)),$$

where

$$\dot{x}^{[M]}(t) = f\left[t, x^{[M]}(t), u_M(t), u_{-M}^*(t)\right], \quad x^{[M]}(t_0) = x_0,$$

$$\dot{x}^*(t) = f\left[t, x^*(t), u_1^*(t), \ldots, u_n^*(t)\right], \quad x^*(t_0) = x_0.$$

The set of all strong equilibrium situations in road sense in the game $\Gamma(x_0, T - t_0)$ we define as $SME\left(\Gamma(x_0, T - t_0)\right)$.

Consider the following vectors:

$$\lambda^{[n,i]} = \left(\lambda_1^{[n,i]}, \ldots, \lambda_i^{[n,i]}, \ldots, \lambda_n^{[n,i]}\right) \in E^n,$$

where $\lambda_j^{[n,i]} = 0, \ j \neq i$ e $\lambda_i^{[n,i]} = 1$.

Theorem. If for any coalition $S \subseteq N, \ S \neq \varnothing$ in a game $\Gamma(x_0, T - t_0)$ exists a number $i_0^S \in S$ and continuously differentiable on $[0, T] \times R$ solution of the following system of differential equations

$$V_t^{[S]}(t,x) + \max_{u_S}\left\{f\left[t, x, u_S(t), \varphi_{-S}^*(t,x)\right]V_x^{[S]}(t,x) + \sum_{i=1}^{n}\lambda_i^{[n,i_0^S]}g_i\left[t, x, u_S(t), \varphi_{-S}^*(t,x)\right]\right\} =$$

$$= V_t^{[S]}(t,x) + f\left[t, x, \varphi_S^*(t,x), \varphi_{-S}^*(t,x)\right]V_x^{[S]}(t,x) + \sum_{i=1}^{n}\lambda_i^{[n,i_0^S]}g_i\left[t, x, \varphi_S^*(t,x), \varphi_{-S}^*(t,x)\right] = 0, \quad (2)$$

$$V^{[S]}(T, x^{[S]}(T)) = \sum_{i=1}^{n}\lambda_i^{[n,i_0^S]}q_i\left[x^{[S]}(T)\right],$$

where for all $S \subseteq N$ maximum in LHS is a unique couple

$$\left\{\varphi_i^*\left(t, x(t)\right) \in U_i, \ i \in N, \ t \in [t_0, T]\right\},$$

where $\varphi_i^*\left(t, x(t)\right) \in U_i, \quad i \in N$ are continuous on $[t_0, T] \times R$ functions, then the couple $\left\{u_i^*(t) = \varphi_i^*\left(t, x(t)\right) \in U_i, \ i \in N, \ t \in [t_0, T]\right\}$ is strong equilibrium in broad sense in the game $\Gamma(x_0, T - t_0)$.

To apply the theorem in an example, consider some properties of the following PDE:

$$\frac{\partial V(t,x)}{\partial t}+\eta_1\left(\frac{\partial V(t,x)}{\partial x}\right)^2+\eta_2\frac{\partial V(t,x)}{\partial x}x+ae^{bt}\frac{\partial V(t,x)}{\partial x}+r(t)=0\ ,\ V(T,x)=\eta_3 x, \tag{3}$$

where a, b, η_1, η_2, η_3 are known constants, $b\neq\eta_2$, $r(t)$ – continuously differentiable function on $[t_0,T]$.

Lemma. Equation (3) has a unique solution $V(t,x)$ on $[t_0,T]$ and:

$$\frac{\partial V(t,x)}{\partial x}=\eta_3 e^{\eta_2(T-t)}.$$

Example

Consider the game $\Gamma(x_0,T-t_0)$, where $N=\{1,2,3\}$, $n=3$,with state dynamics:

$$\dot{x}(t)=ax+b_1u_1+b_2u_2+b_3u_3,\quad x(t_0)=x_0. \tag{4}$$

Player $1\in N$ tends to maximize the following functional:

$$J_{\{1\}}\left[x_0,u_1,u_2,u_3\right]=\int_{t_0}^{T}\left[-u_1^2-u_2^2-u_3^2+u_1x+u_2x+u_3x-\frac{3}{4}x^2+r^{[1]}(t)\right]dt+x(T), \tag{5}$$

the aim of player $2\in N$ is functional $J_{\{2\}}\left[x_0,u_1,u_2,u_3\right]$:

$$J_{\{2\}}\left[x_0,u_1,u_2,u_3\right]=\int_{t_0}^{T}\left[u_1^2-u_2^2-u_3^2-u_1x+u_2x+u_3x-\frac{1}{4}x^2+r^{[2]}(t)\right]dt+x(T), \tag{6}$$

and the aim of player $3\in N$ is functional $J_{\{2\}}\left[x_0,u_1,u_2,u_3\right]$:

$$J_{\{3\}}\left[x_0,u_1,u_2,u_3\right]=\int_{t_0}^{T}\left[-3u_1^2+u_2^2-u_3^2+3u_1x-u_2x+u_3x-\frac{3}{4}x^2+r^{[3]}(t)\right]dt+x(T), \tag{7}$$

where $r^{[1]}(t)$, $r^{[2]}(t)$, $r^{[3]}(t)$, $t\in[t_0,T]$ – continuously differentiable functions. In the game (4)-(7) strong equilibrium *SME* is found.

References

[1] Courant, R. (1962). *Partial differential equations*. New York - London.: Intersci. Publ.

[2] Grauer, L. V. and Petrosjan, L.A. (2002). Strong Nash Equilibrium in Multistage Games. *International Game Theory Review*, vol. 4(3), pp. 255 -264.

[3] Isaacs, R. (1965). *Differential games*. New York, London, Sydney: John Wiley and sons Inc.

[4] Zenkevich, N.A., Petrosyan, L.A. and Yeung D.W.K. (2009). *Dynamic games and management applications*. SPb.: GSOM.

Index

Game Theory and Management

Editors: Leon A. Petrosyan, Nikolay A. Zenkevich

Abstracts

Authorized: 21.06.2010. Author's sheets: 12,0
125 copies
Publishing House of the Graduate School of Management, SPSU
Volkhovsky per. 3, St. Petersburg, 199004, Russia
tel. +7 (812) 323 84 60
www.gsom.pu.ru